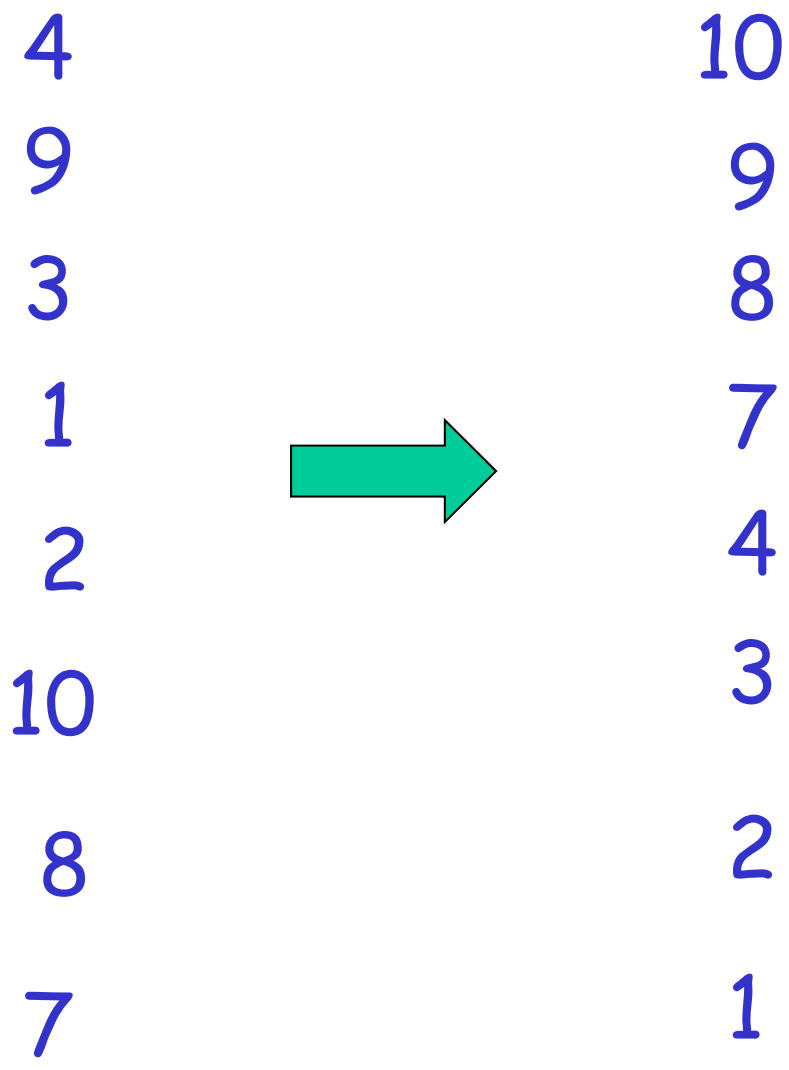
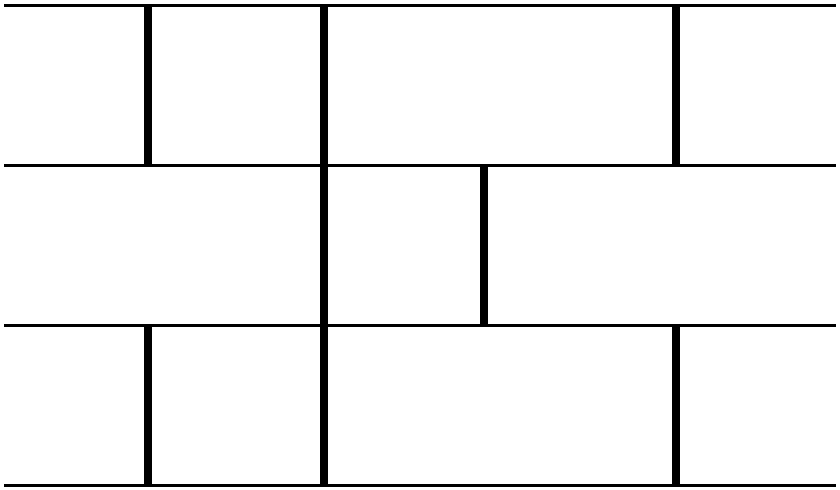
Mreže za sortiranje

Sortiranje

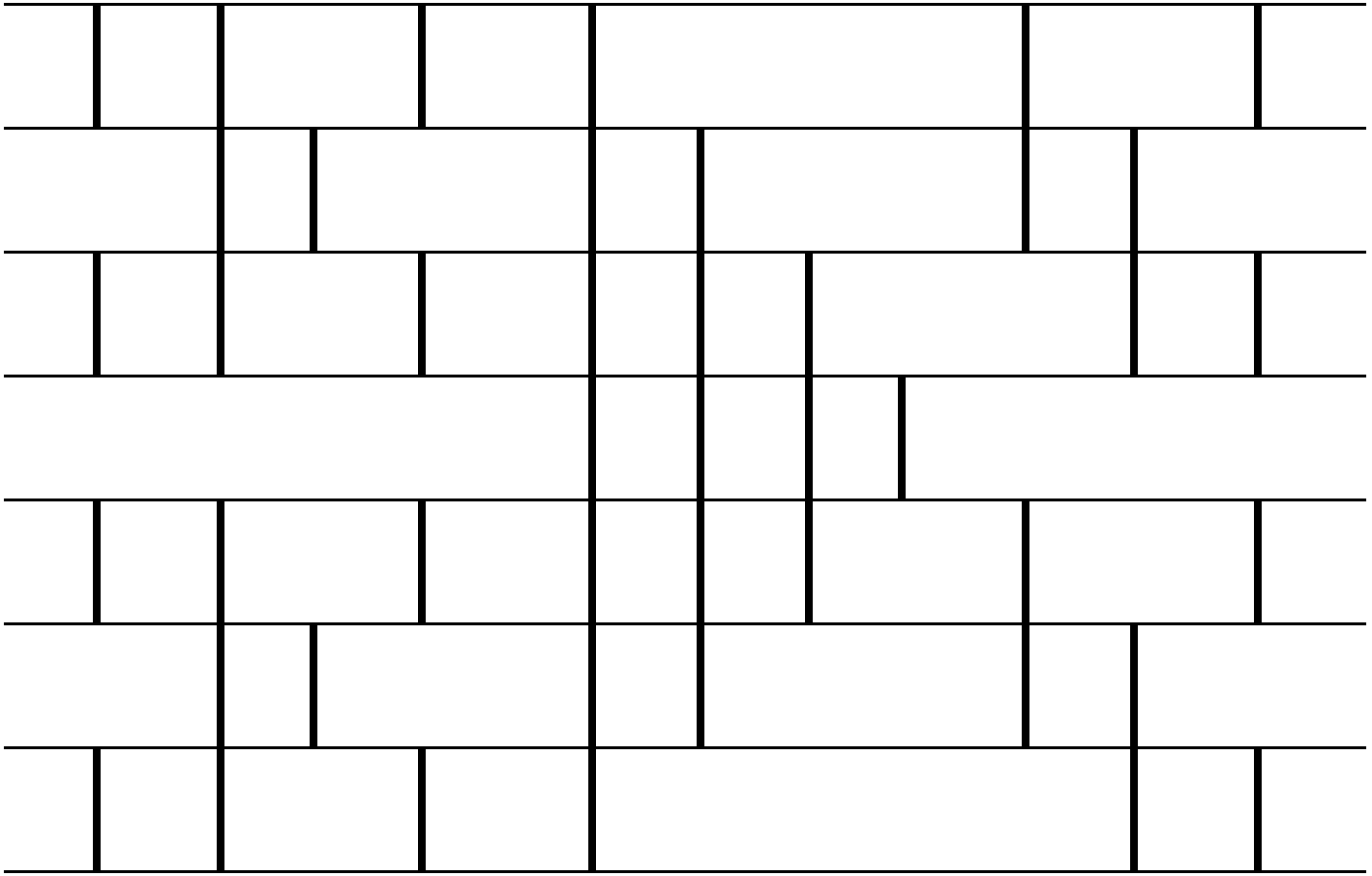


Mreža za sortiranje



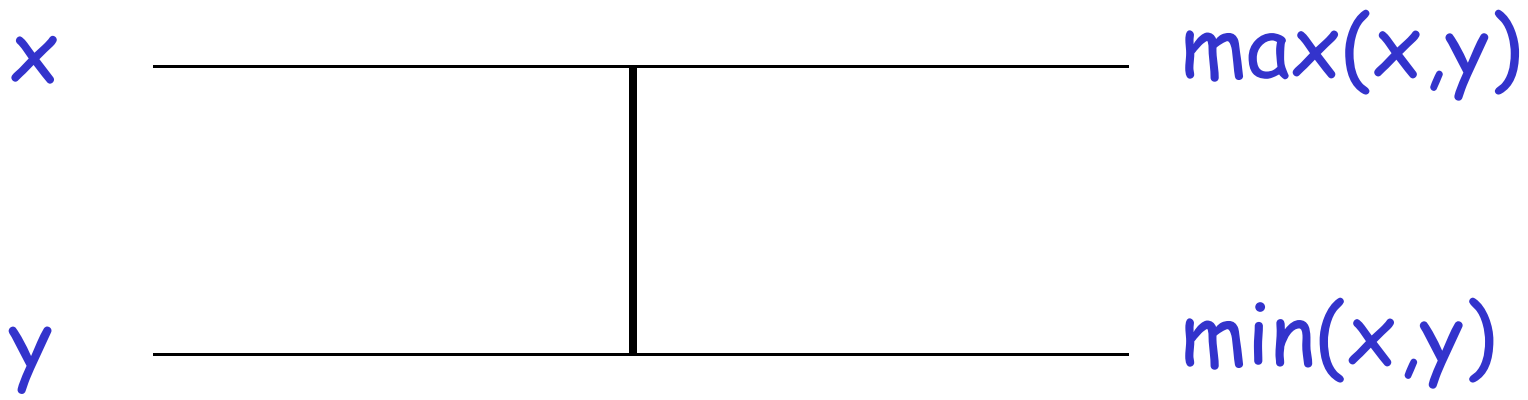
(Mreža za bitonik sortiranje)

Mreža za sortiranje



(Mreža za bitonik sortiranje)

Komparator



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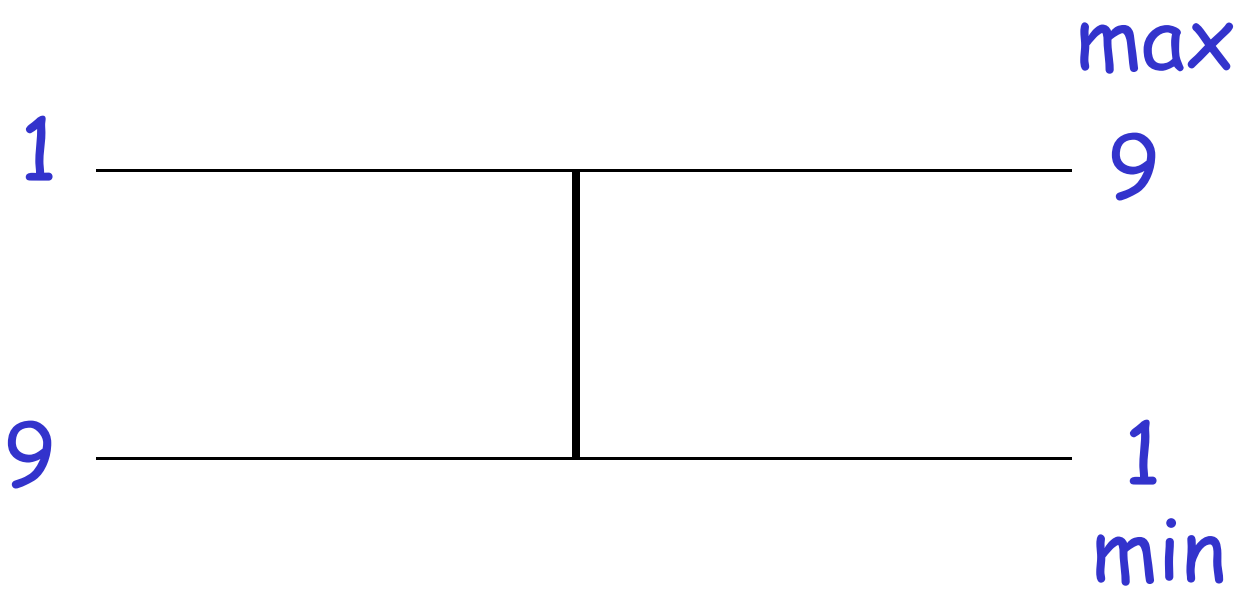

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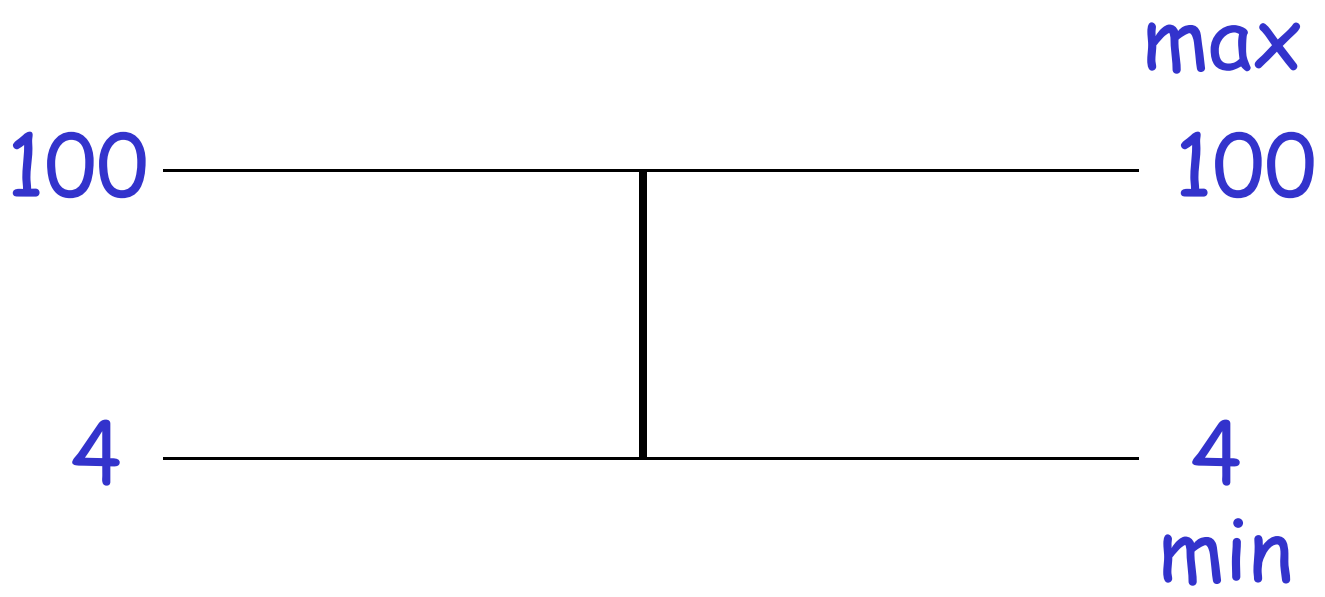
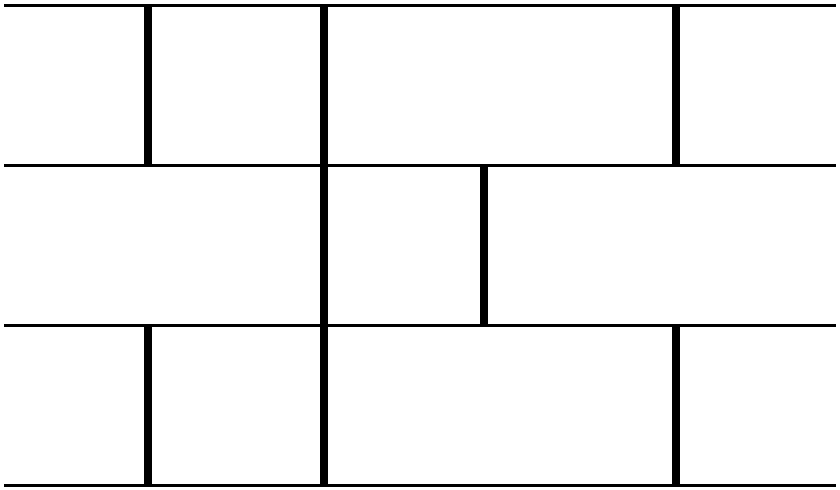
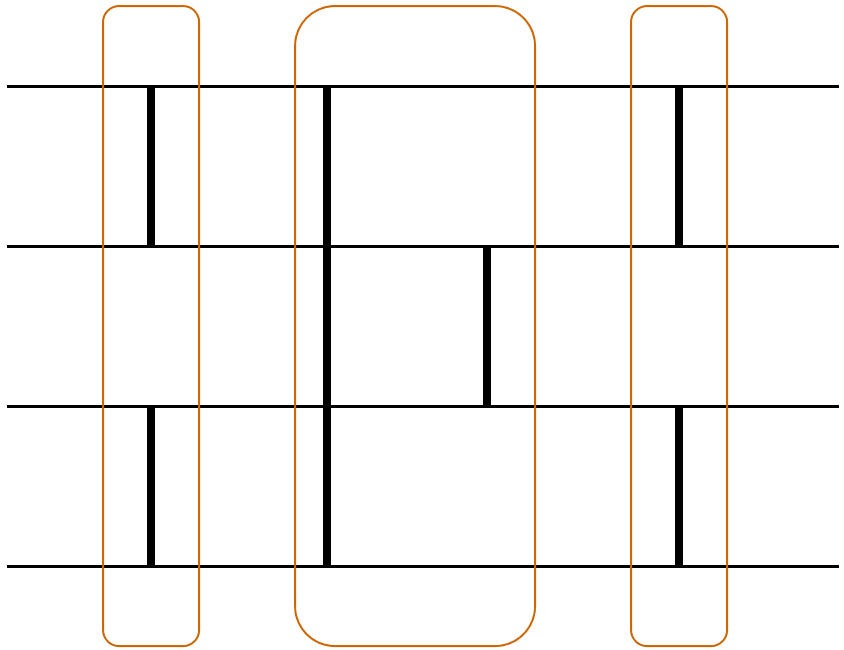




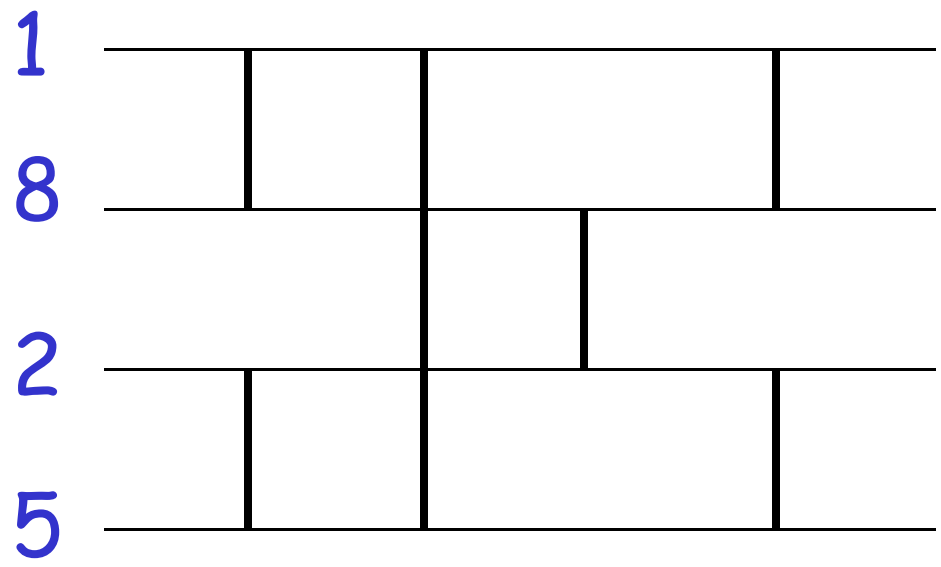
Nivoi

1 2 3



Širina = 4

Dubina = 3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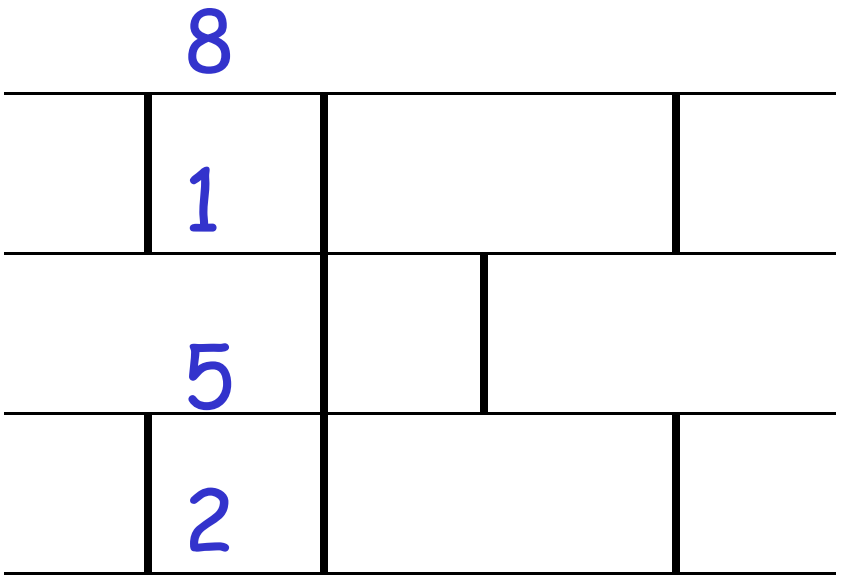

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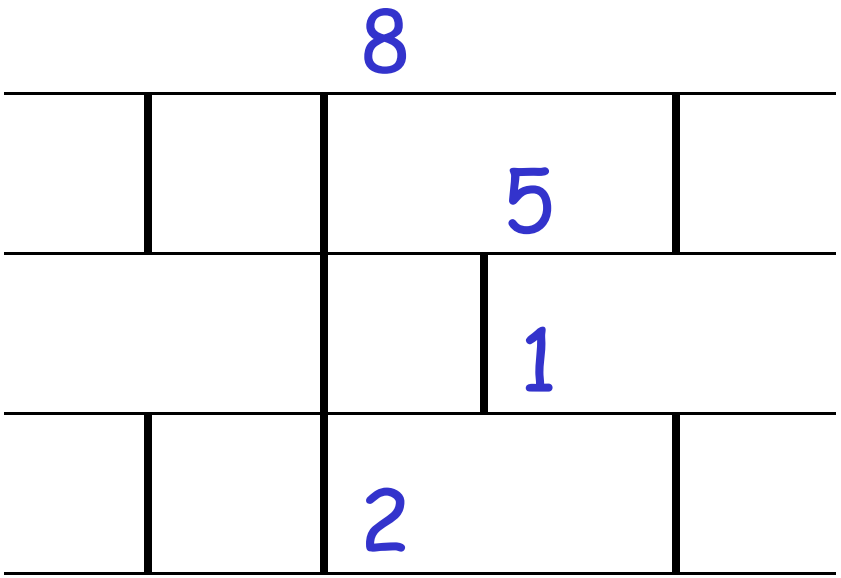

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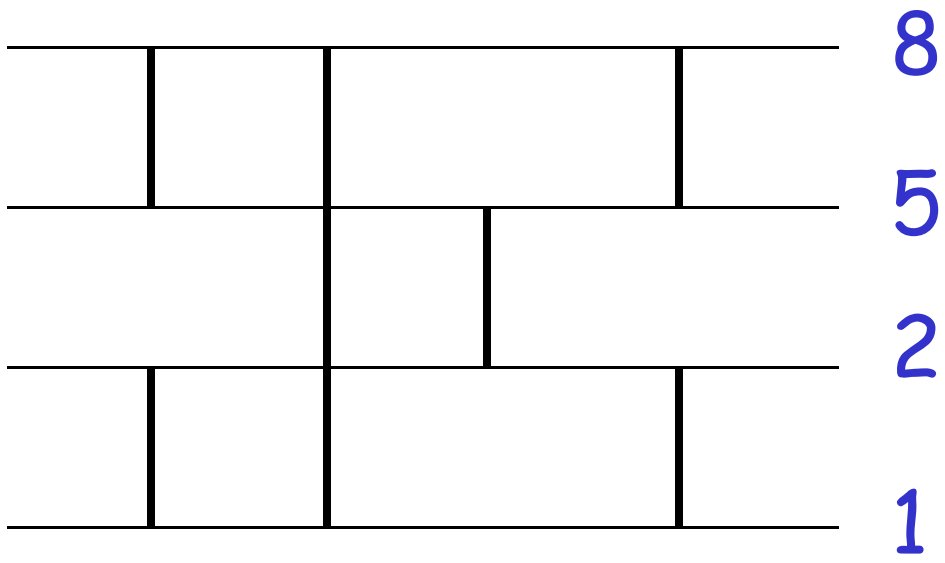

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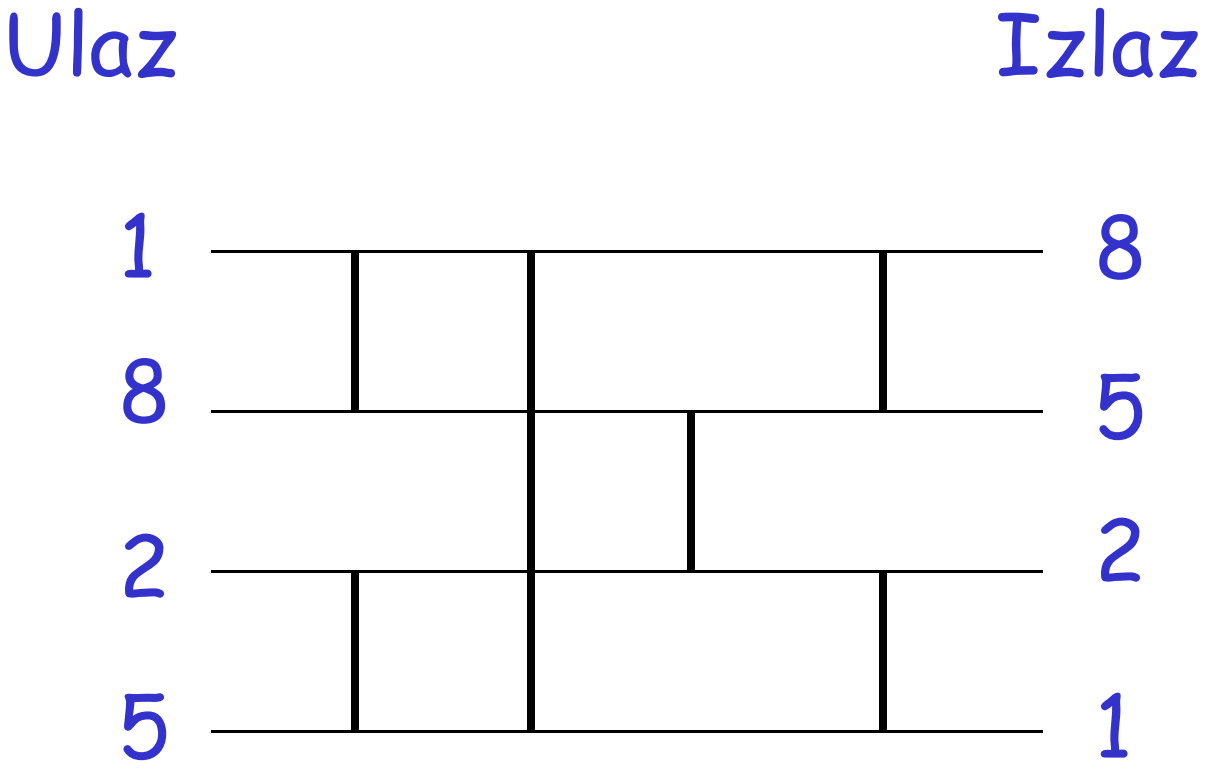
Ulaz
Izlaz
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2
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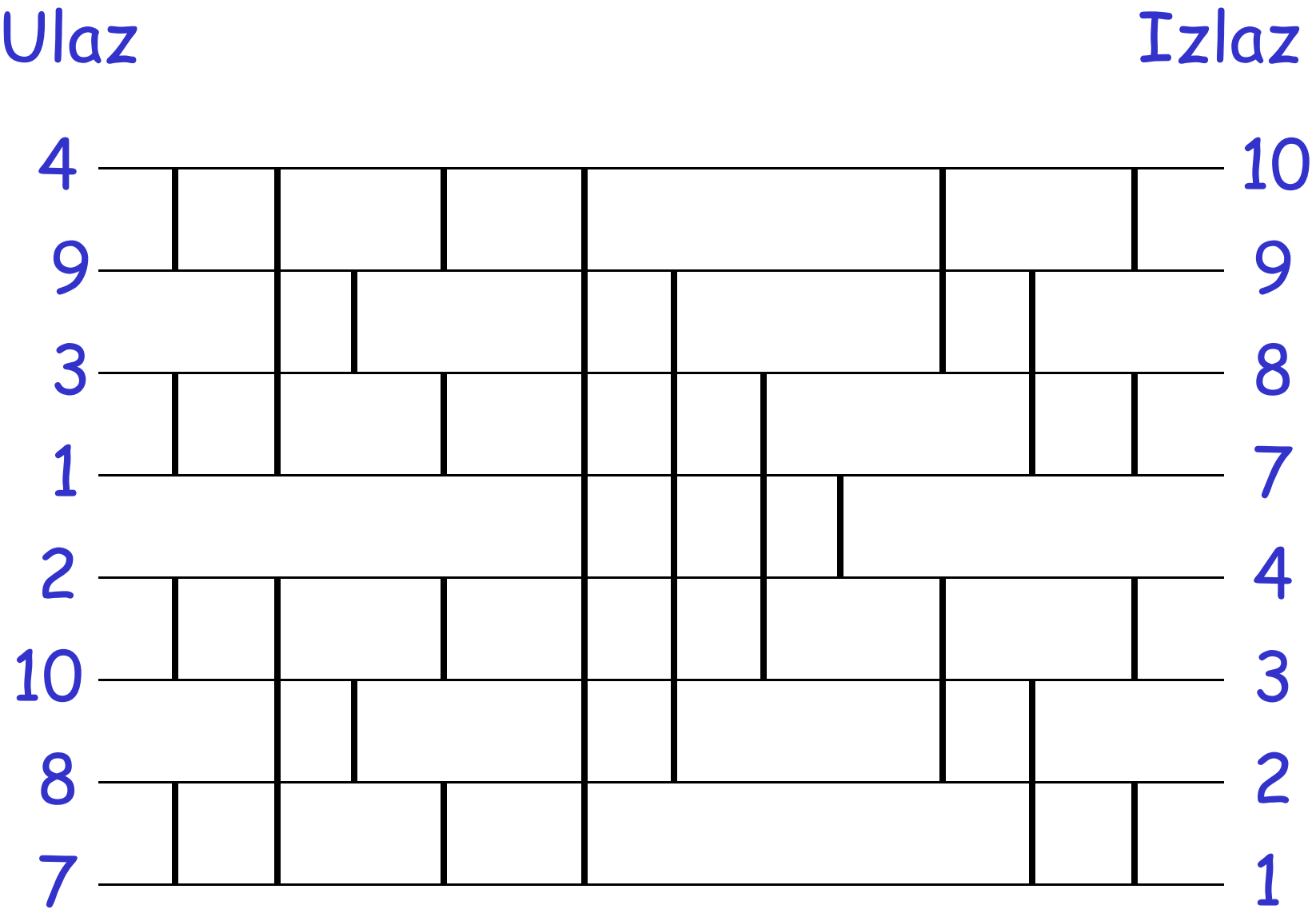
8
5
2
1


Ulaz
Izlaz
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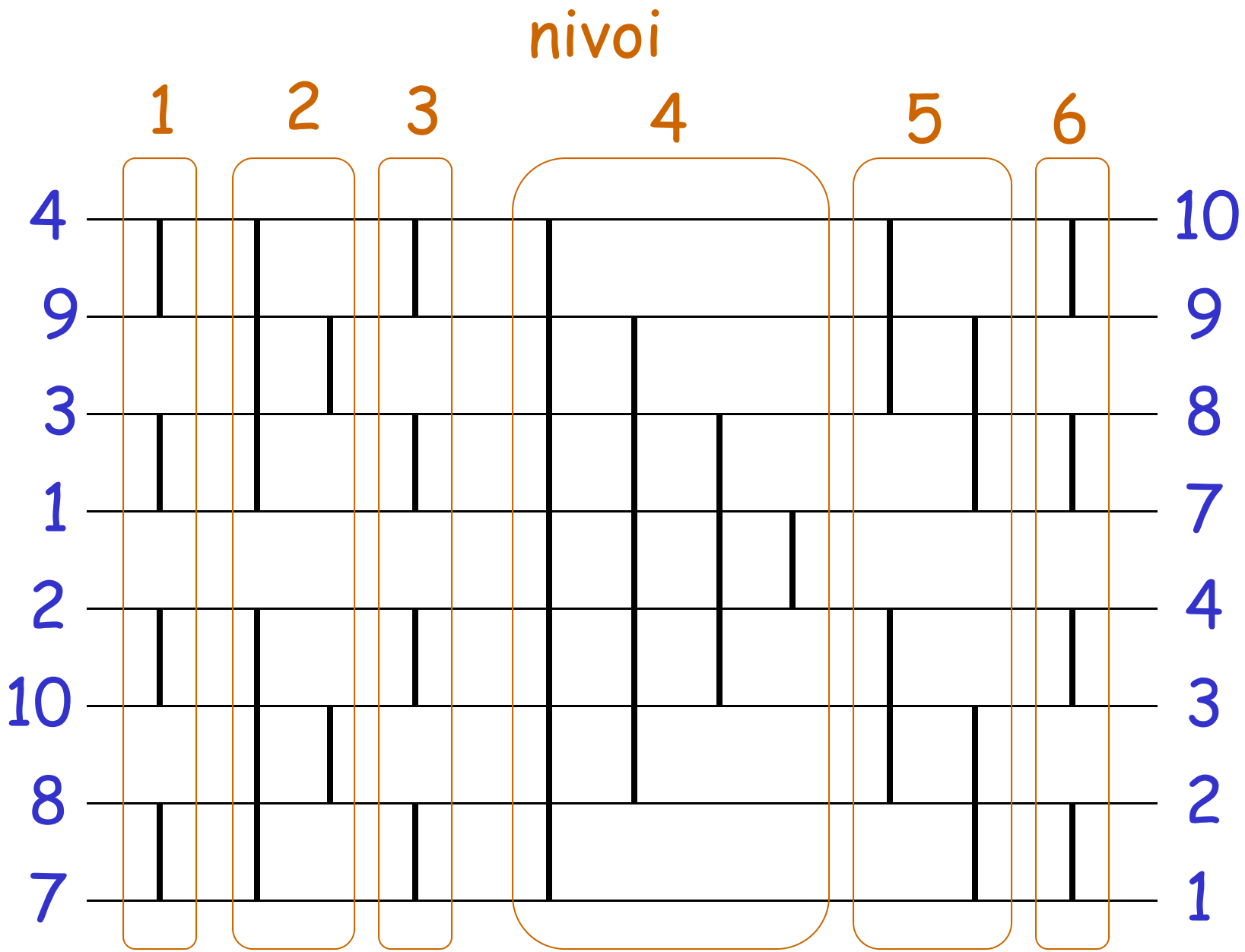
nivoi
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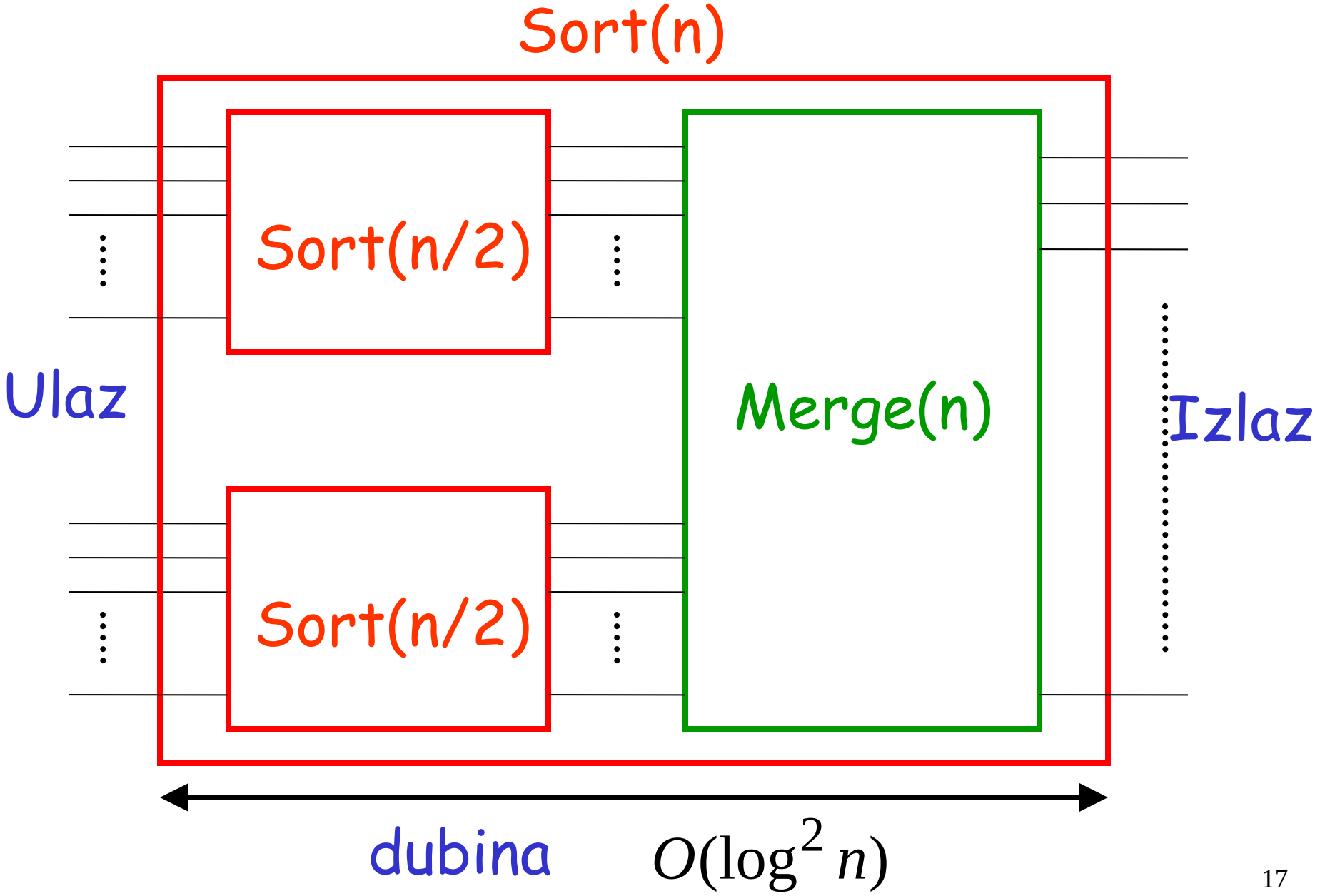
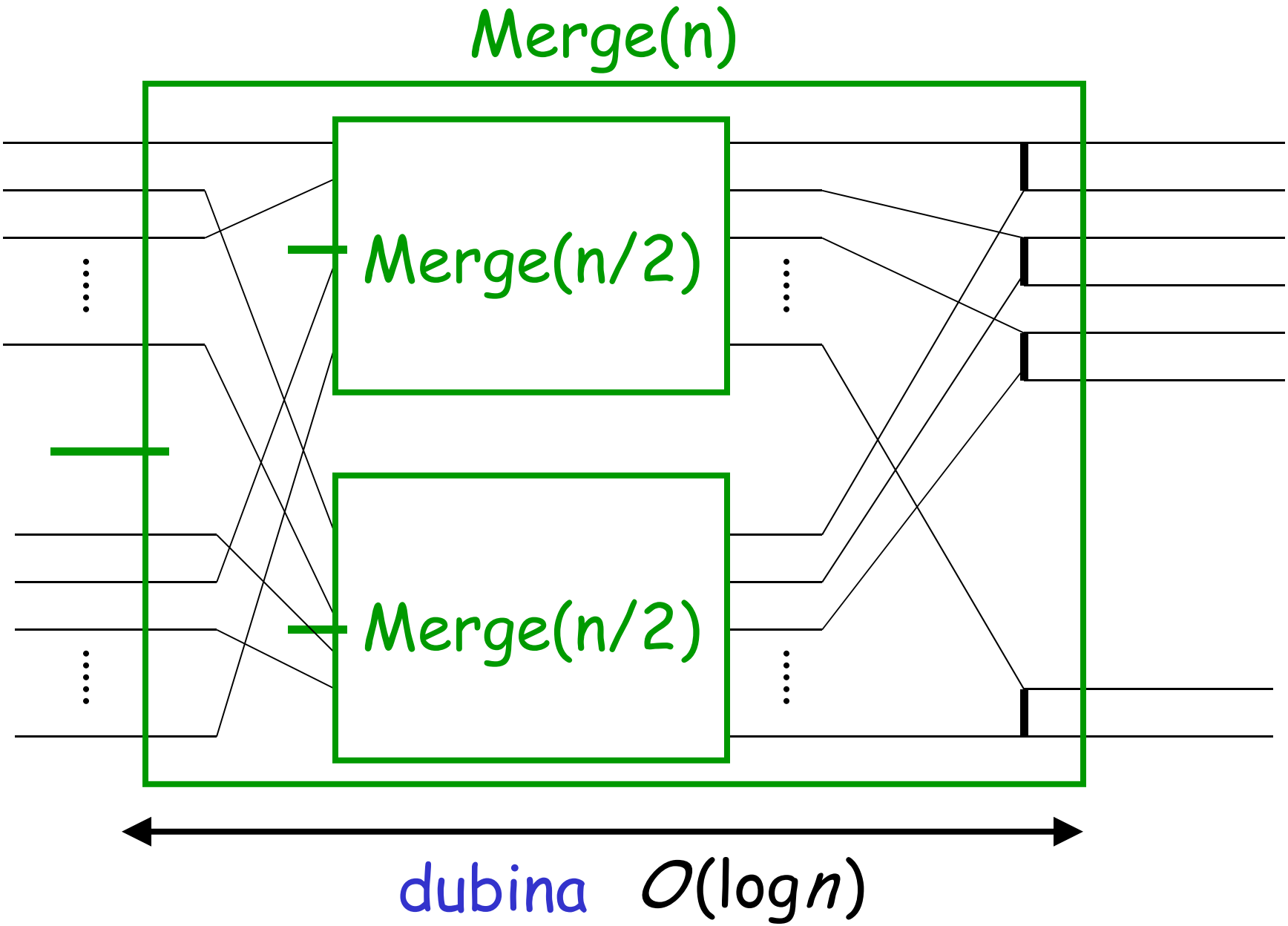
2
1

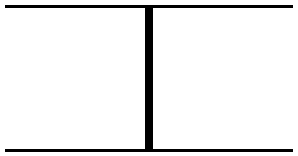
Rekurzivna konstrukcija Mreža za bitonik sortiranje

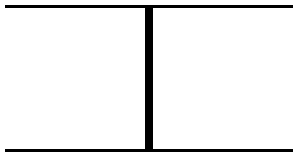


Rekurzivna konstrukcija Mreže za spajanje



Osnova za indukciju

Sort(2) 

Merge(2) 

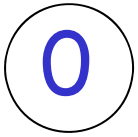
Širina n

Dubina mreže za sortiranje: $O(\log^2 n)$

Širina spajača	n	$\frac{n}{2}$	$\frac{n}{4}$	2
Ukupna dubina	$\log n + \log\frac{n}{2} + \log\frac{n}{4} + \ldots + \log 2 = O(\log^2 n)$			

Mreža za brojanje

Problem brojanja



Deljena promenljiva

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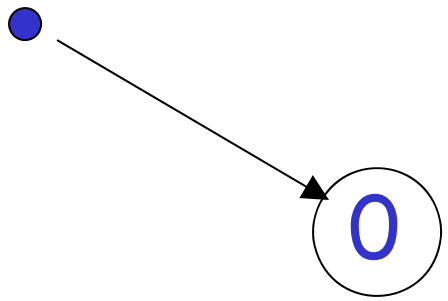

Deljena promenljiva

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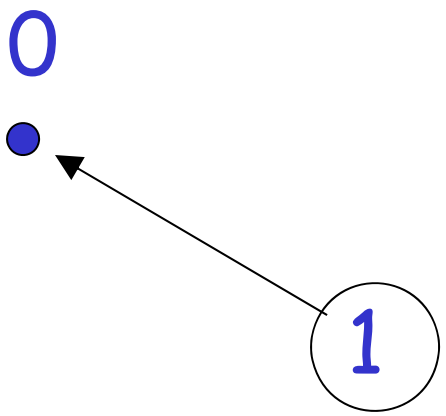
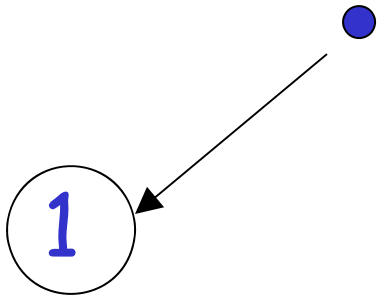


Deljena promenljiva

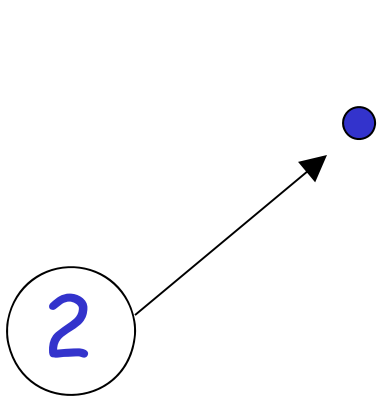
Deljena promenljiva

Zahtev za inkrement



Deljena promenljiva

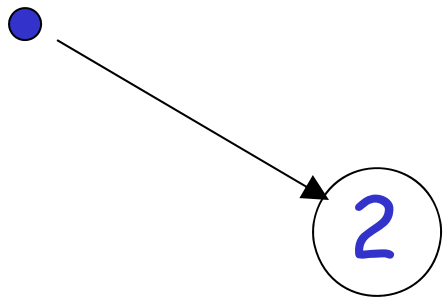
Zahtev za inkrement



Deljena promenljiva

Deljena promenljiva

Zahtev za inkrement



Deljena promenljiva

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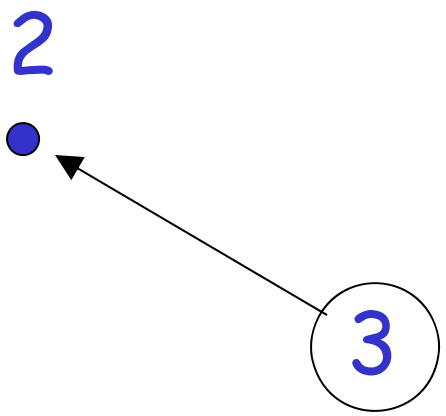
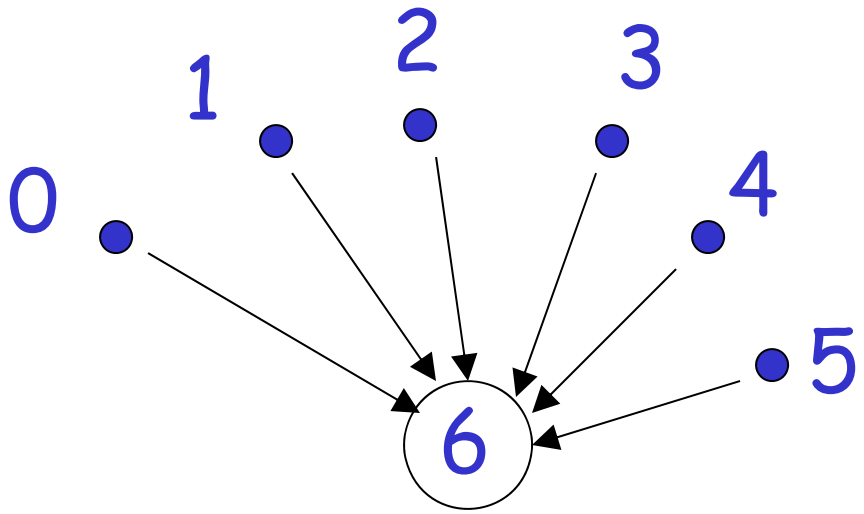


Deljena promenljiva

Sekvencijalno usko-grlo



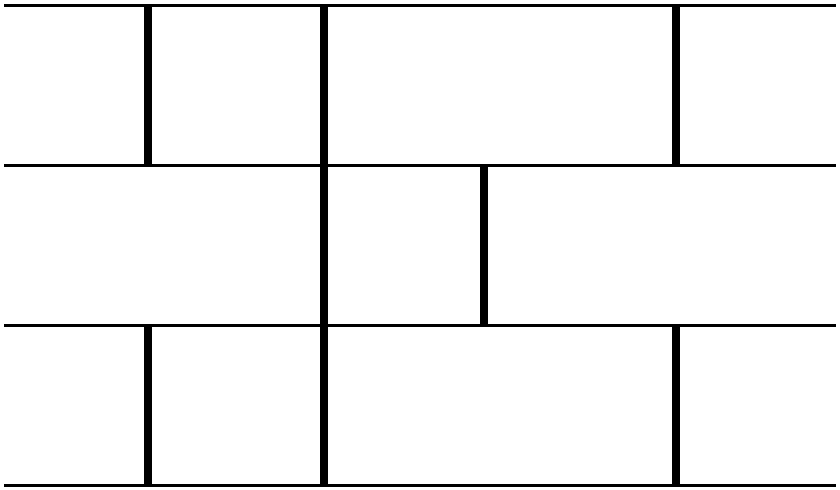
Deljena promenljiva

Zahtevi moraju biti serializovani

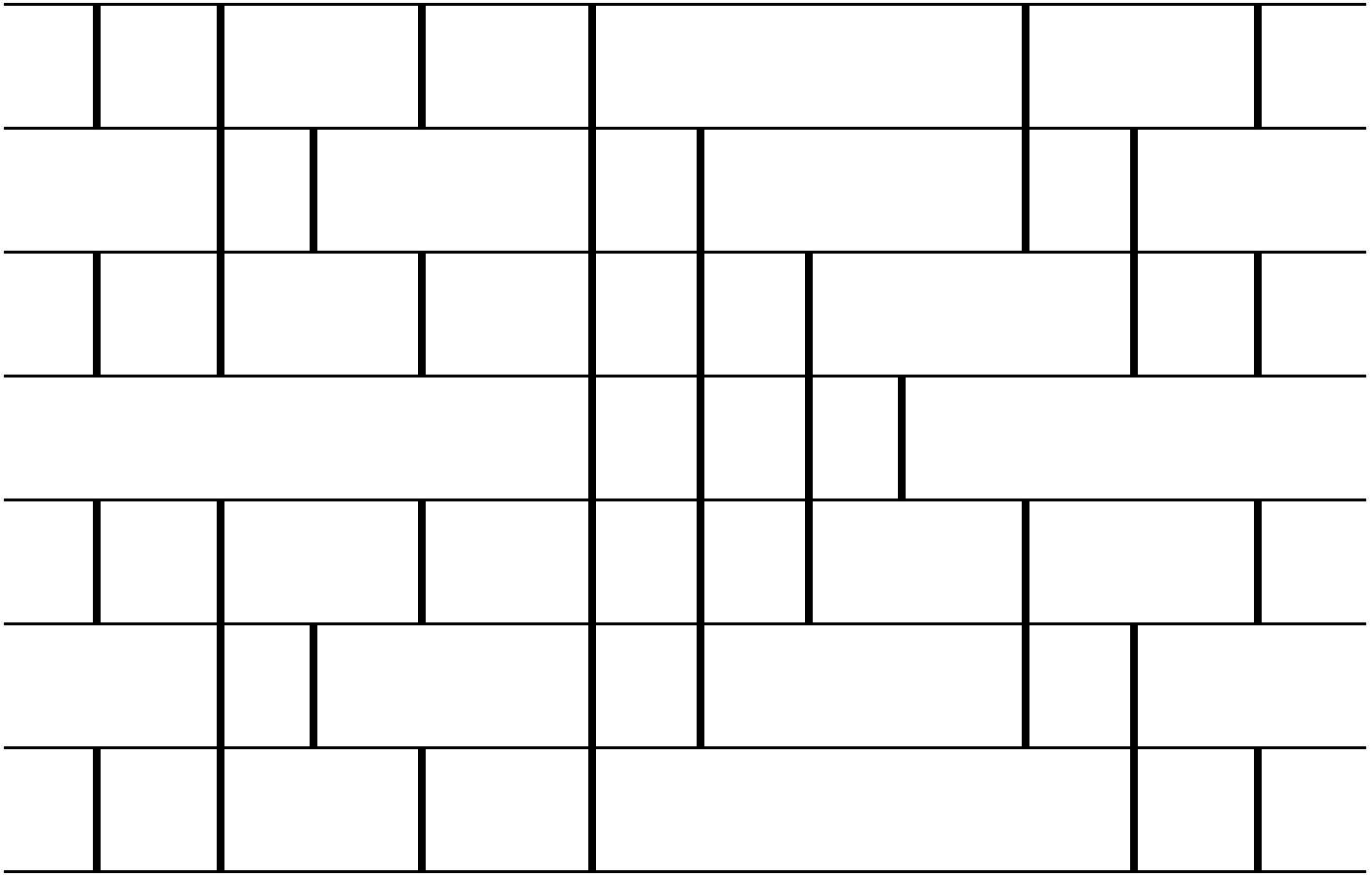
Mreža za brojanje

Ulazi

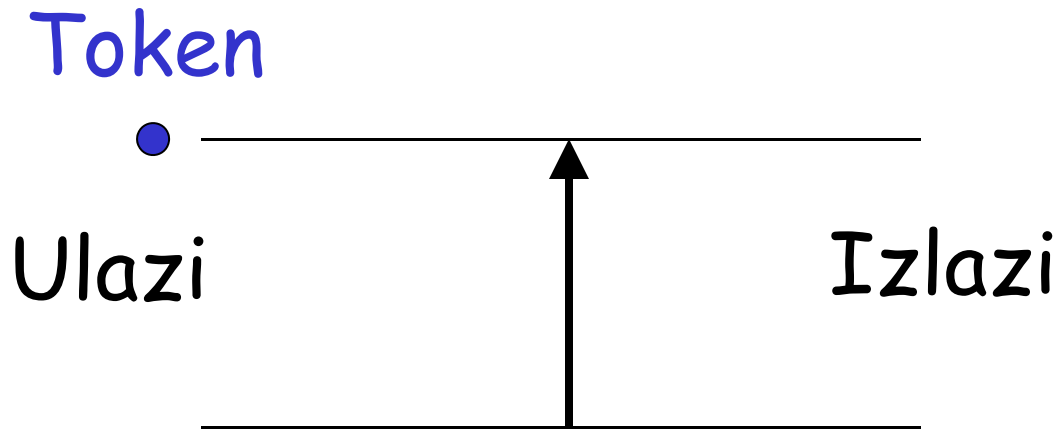
Izlazi



Mreža za brojanje



Balanser



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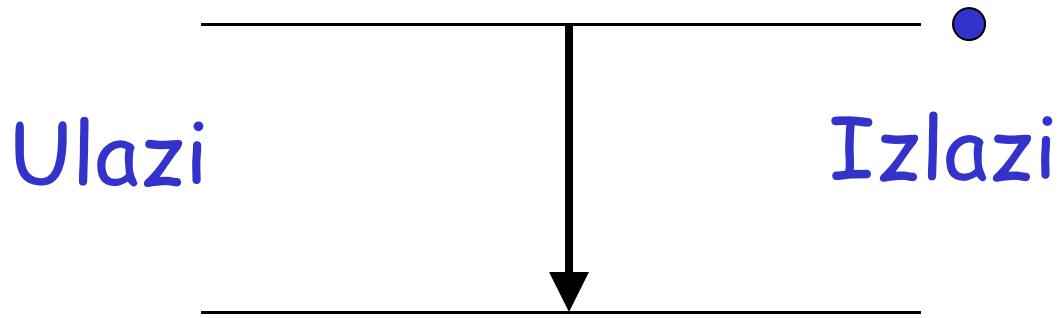

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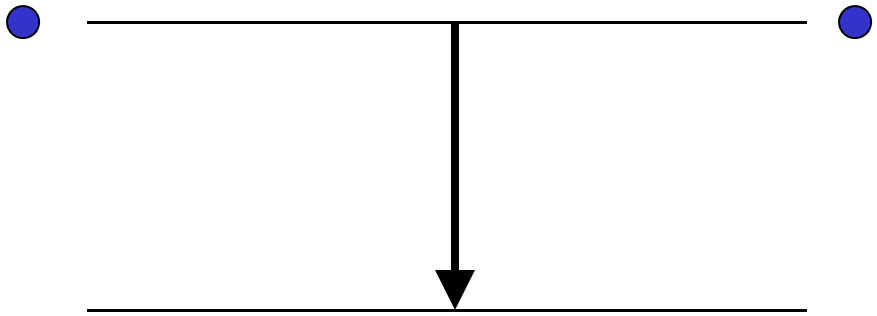

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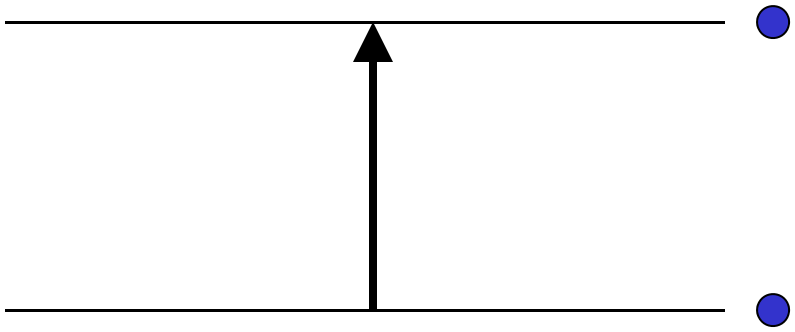

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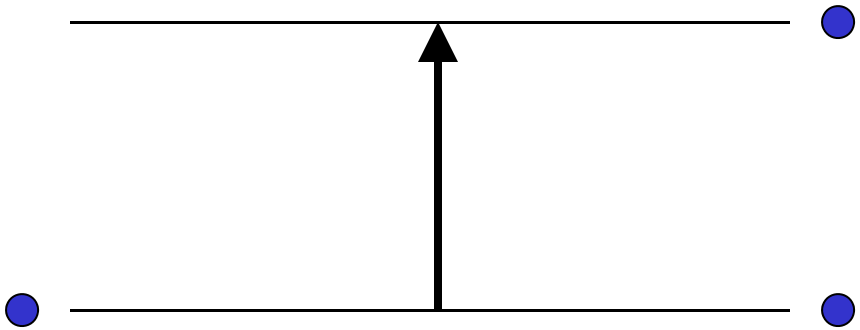

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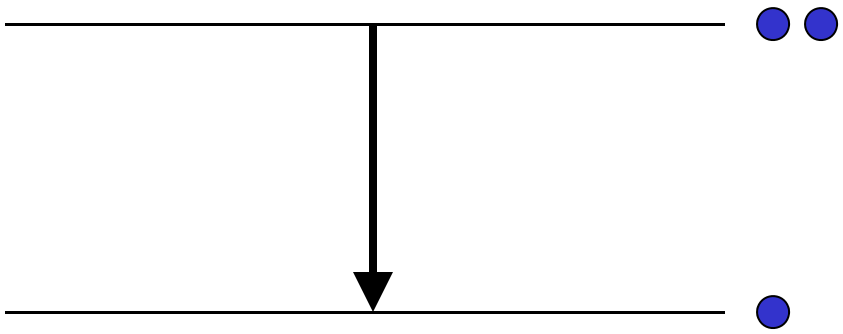

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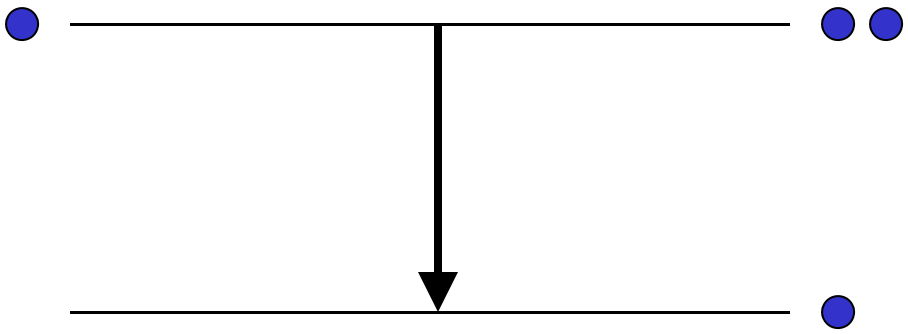

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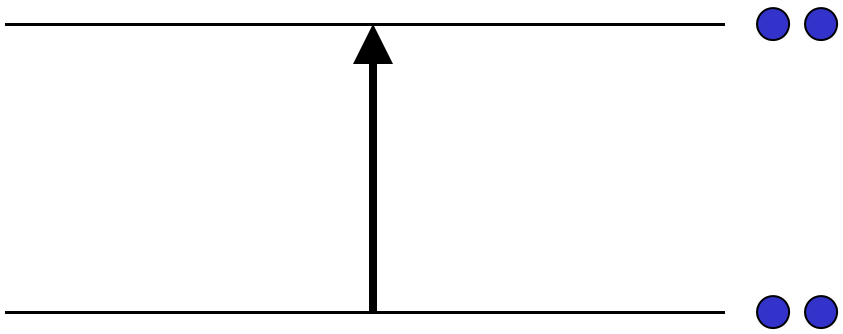

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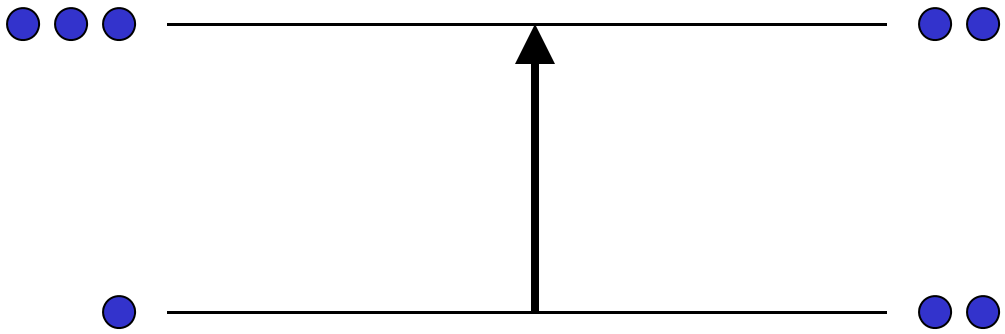

Svi tokeni zajedno

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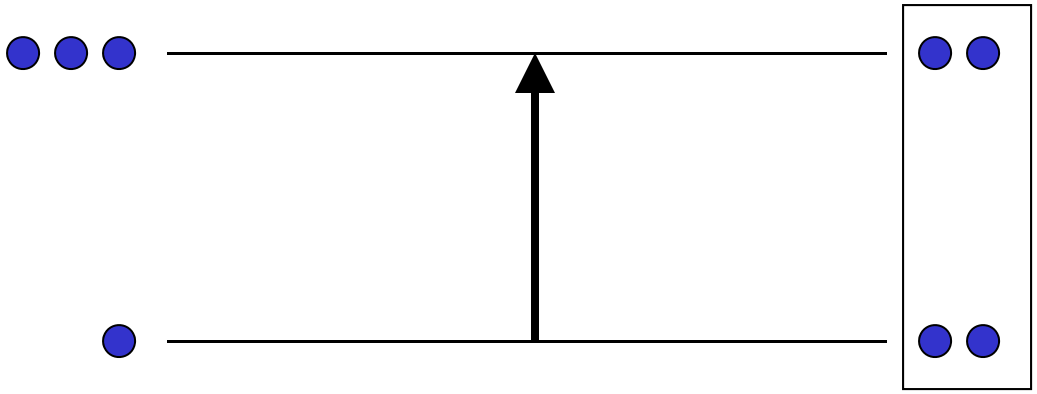

Osobina koraka

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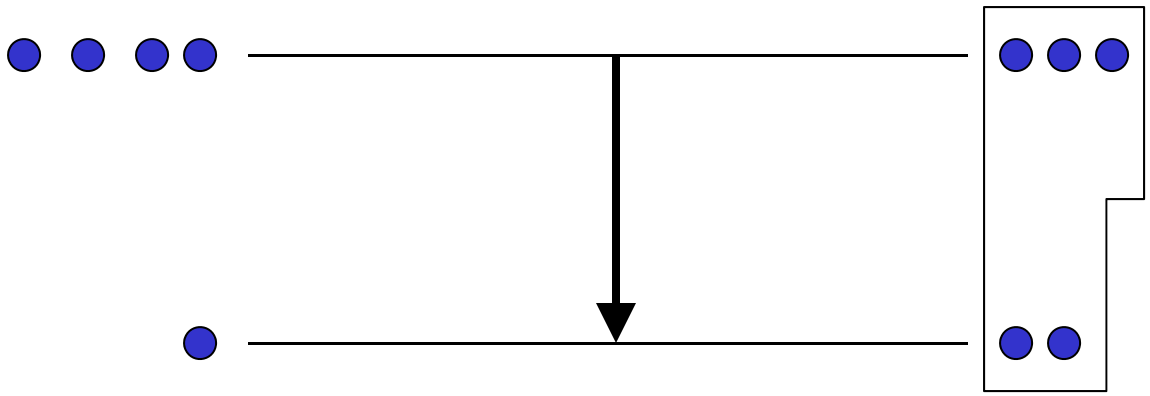
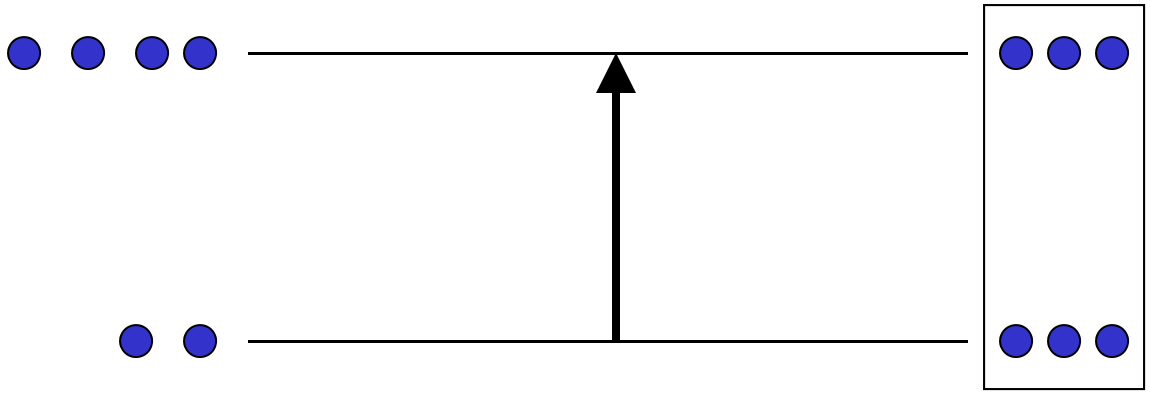


Osobina koraka

Sledeći primer

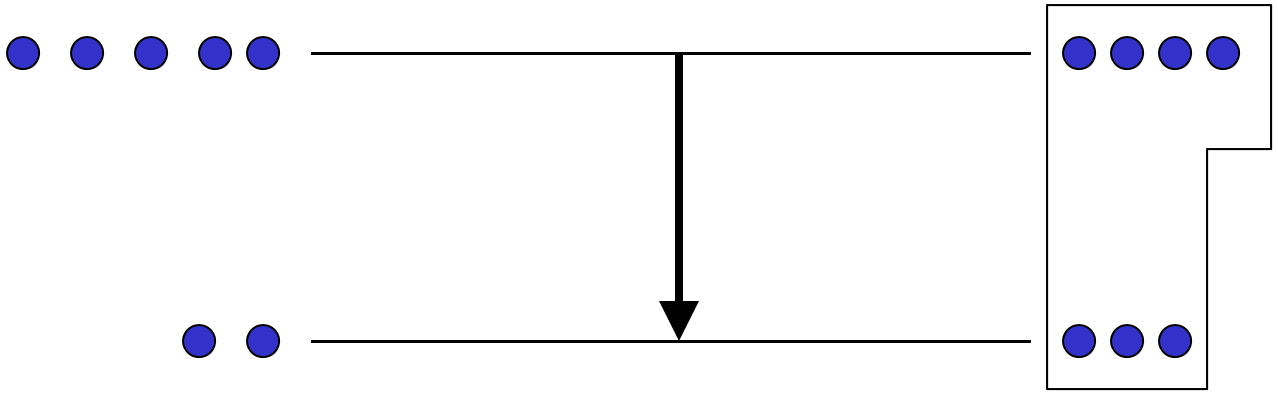
Balanser



Osobina koraka

Sledeći primer

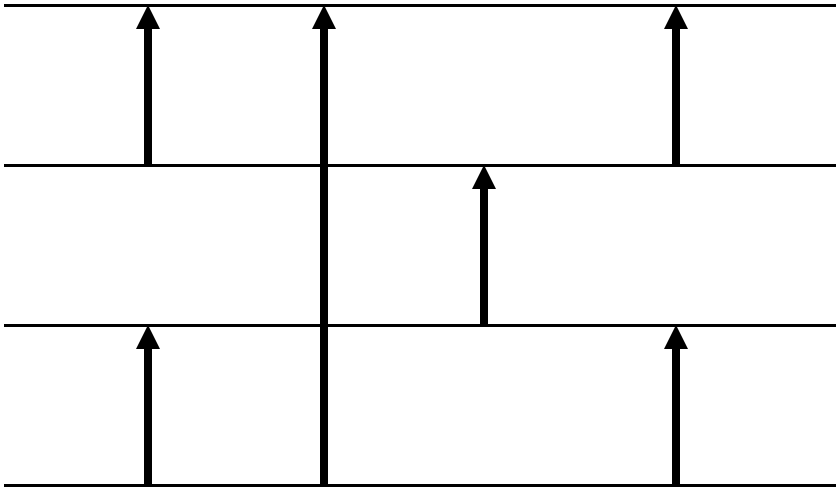
Balanser



Osobina koraka

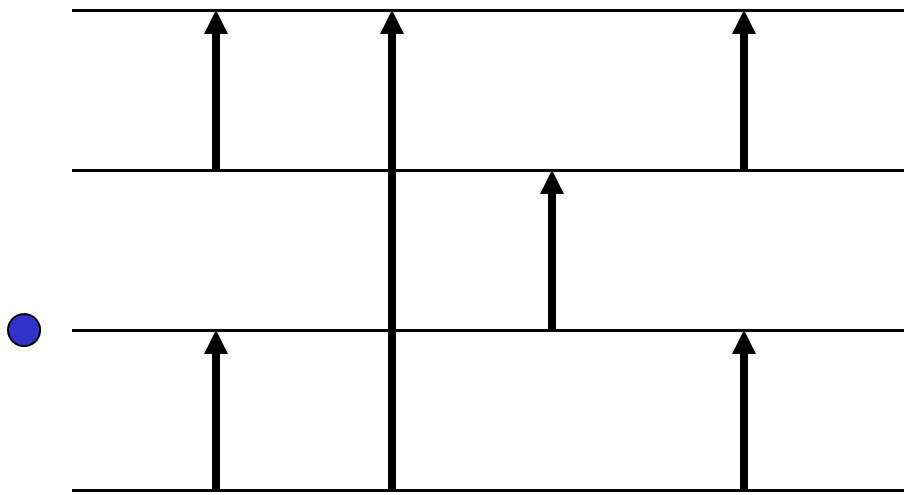
Sledeći primer

Mreža za brojanje



(Mreža za bitonik brojanje)

Mreža za brojanje



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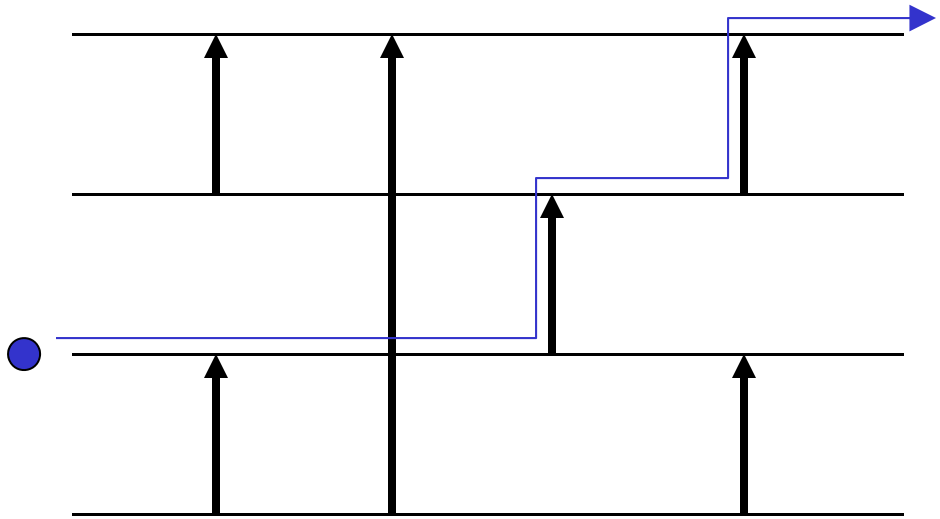

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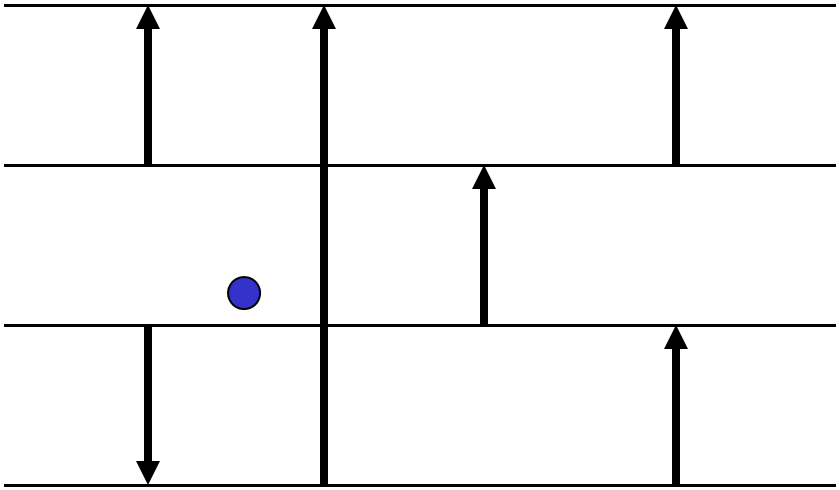

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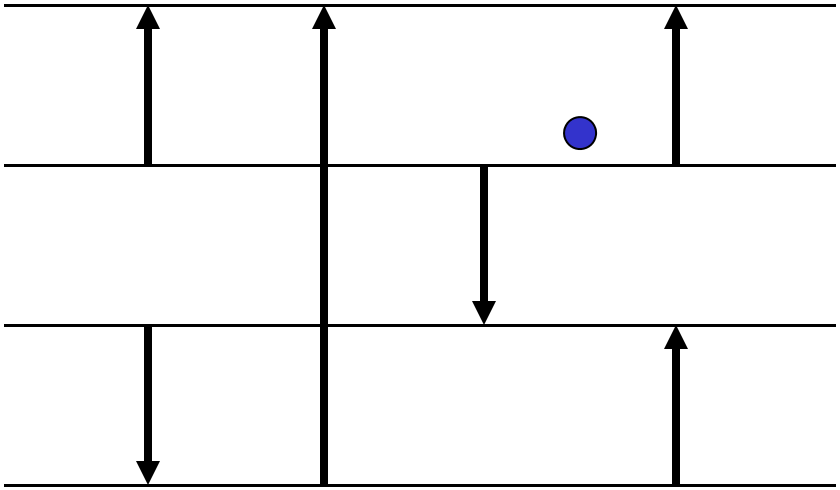

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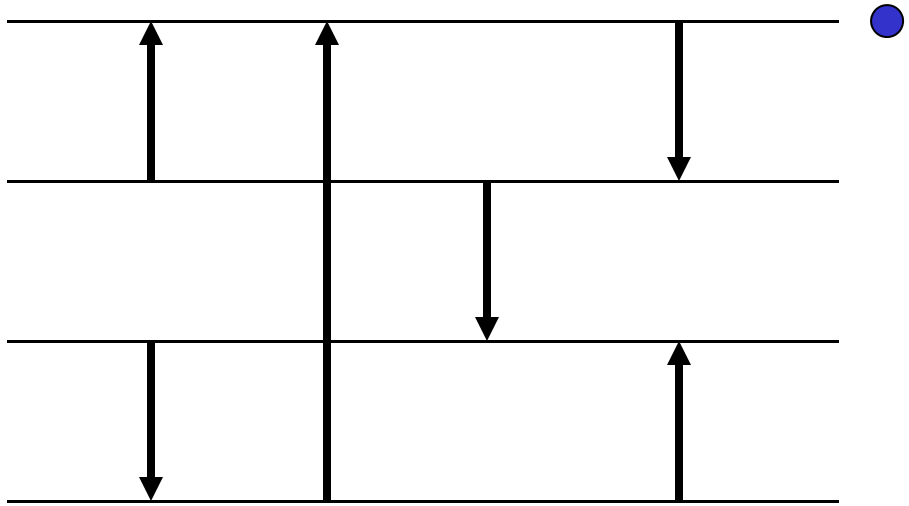

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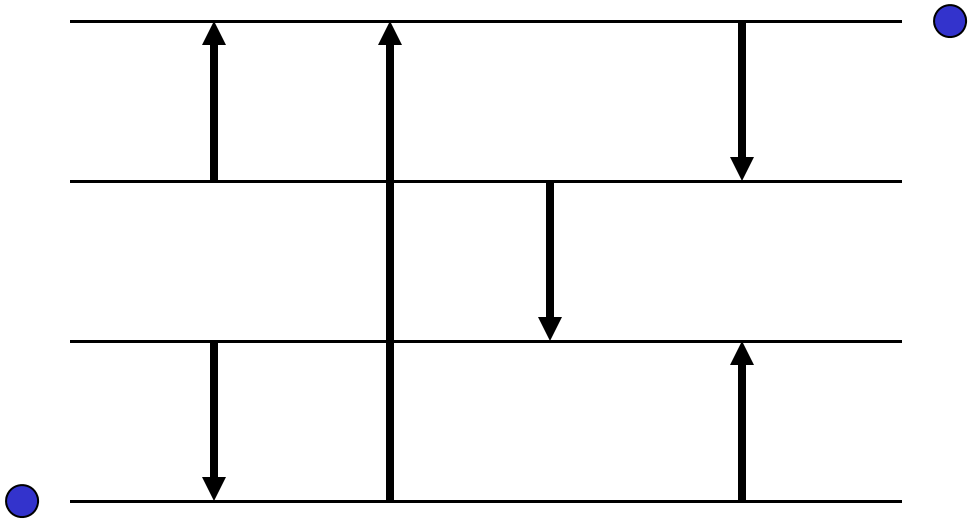

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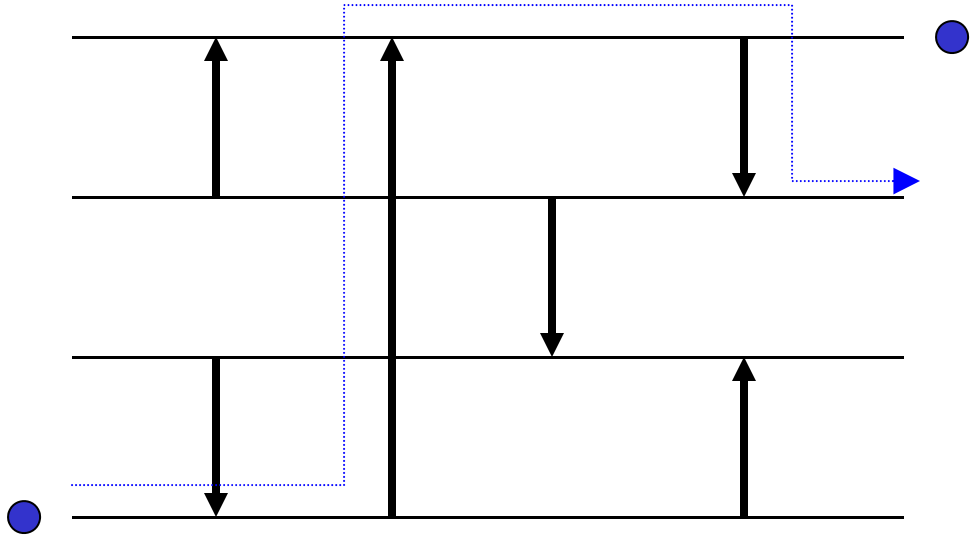

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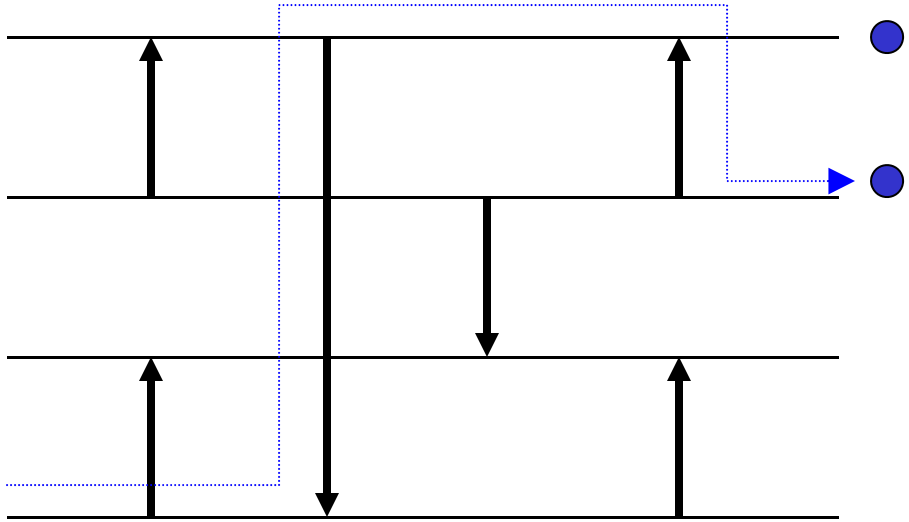

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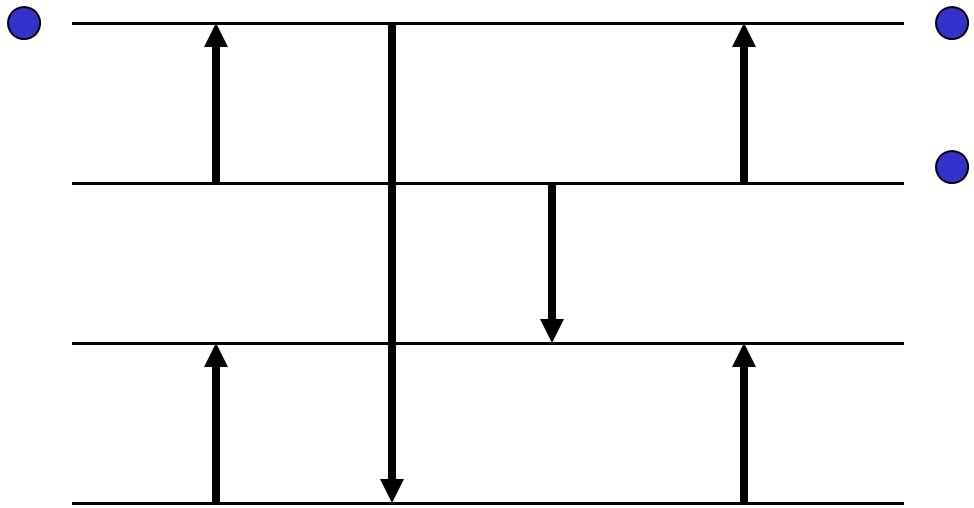

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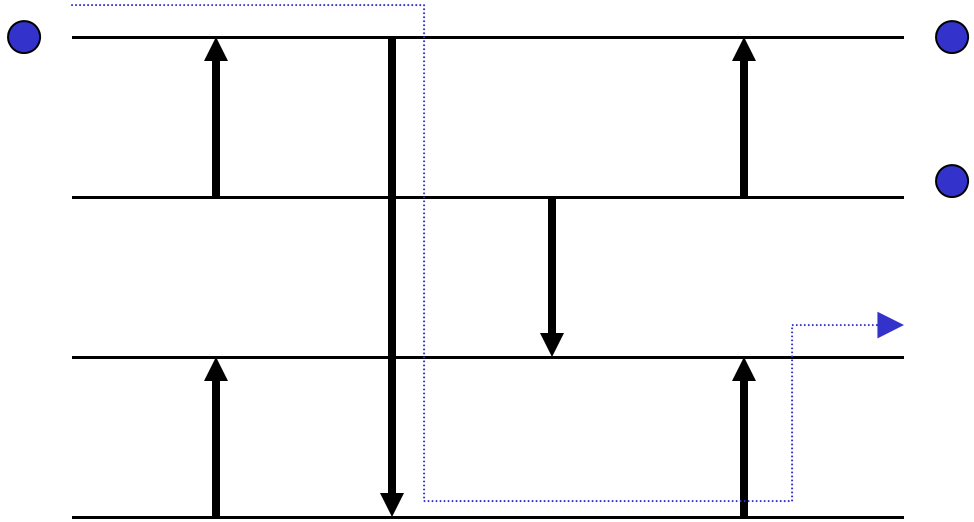

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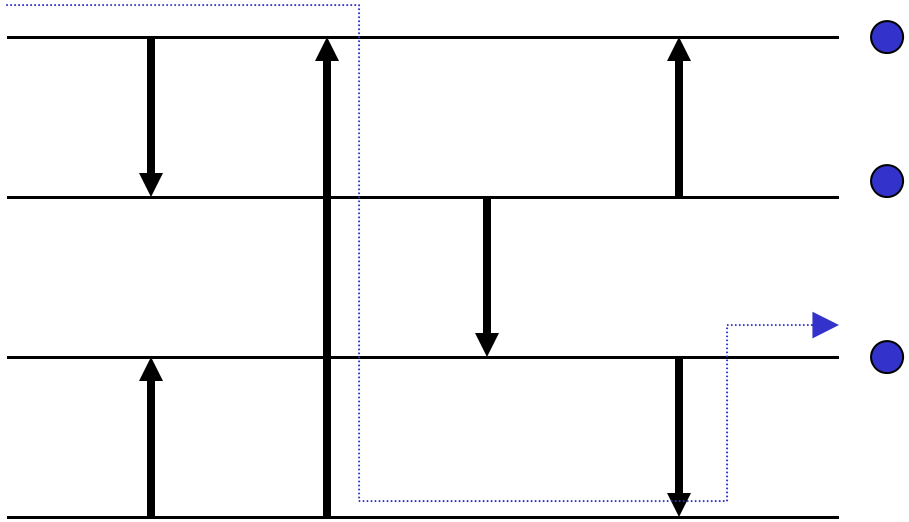

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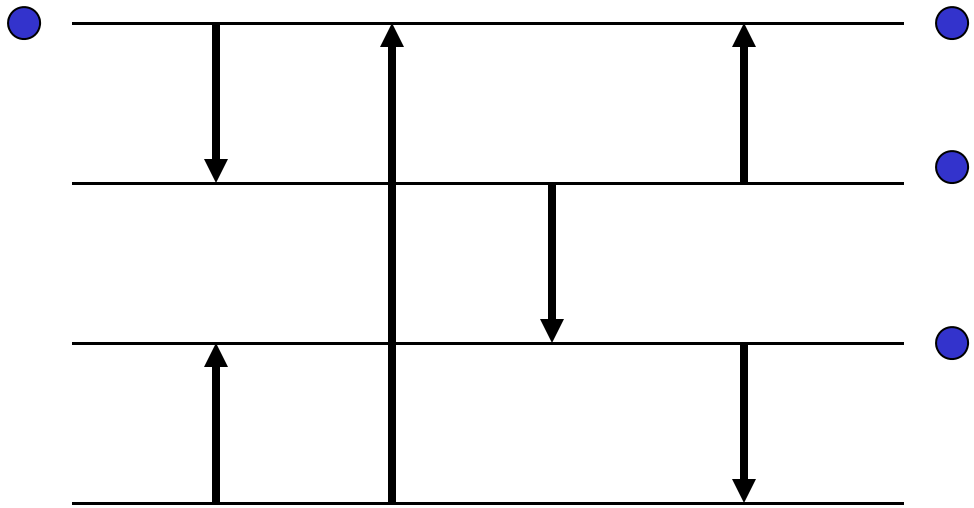
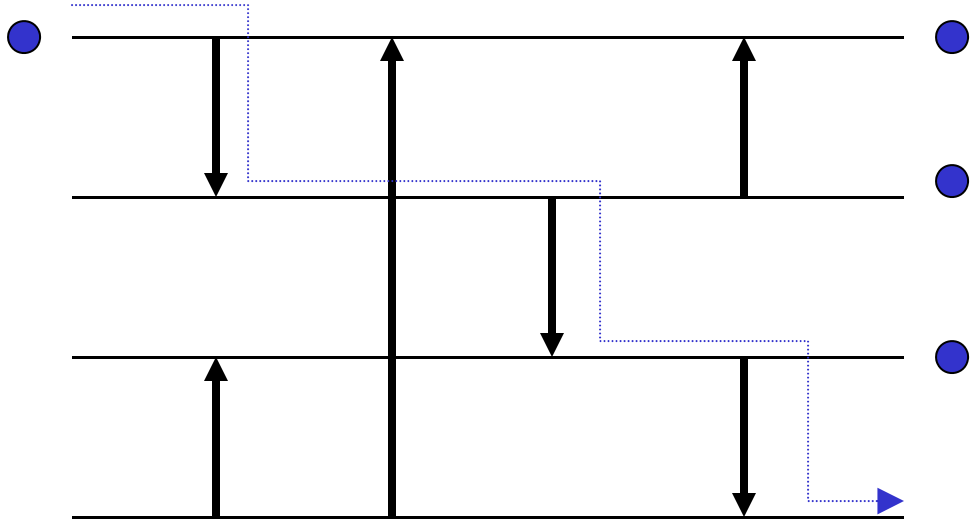


Mreža za brojanje



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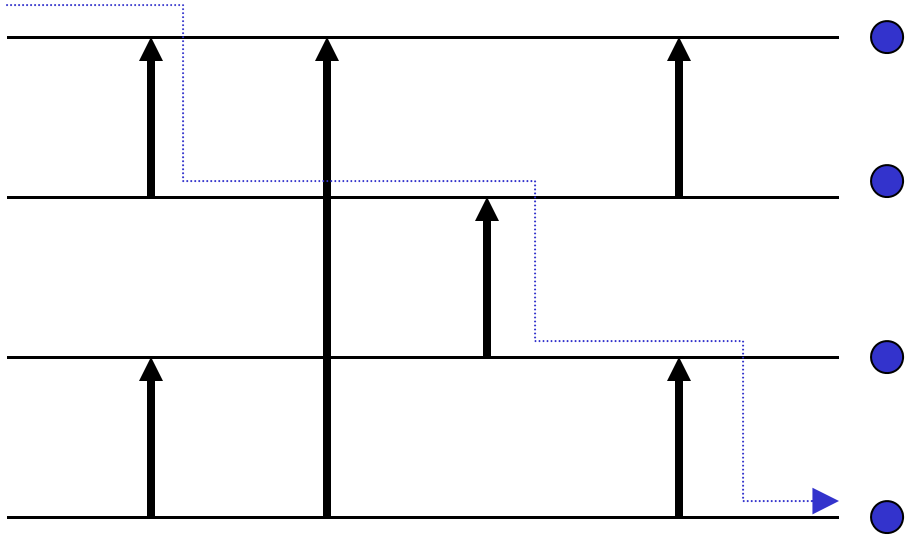

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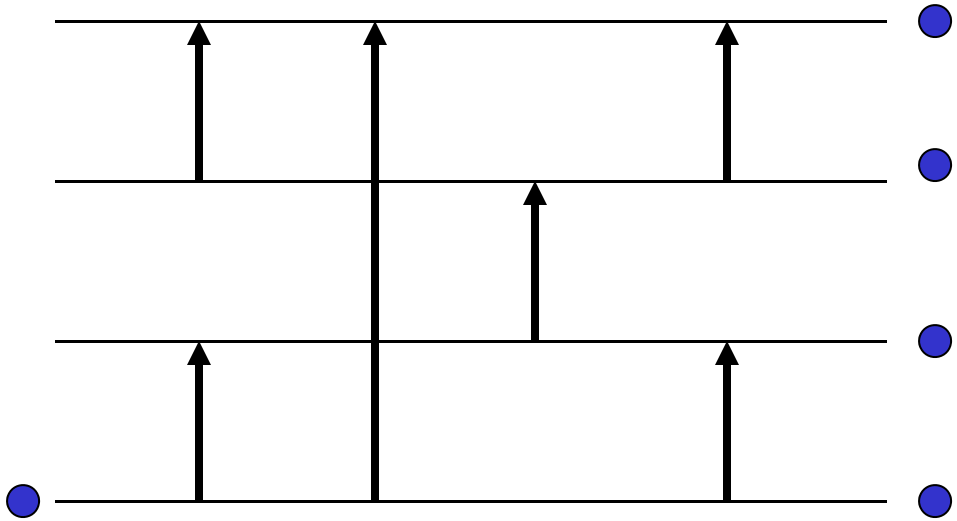

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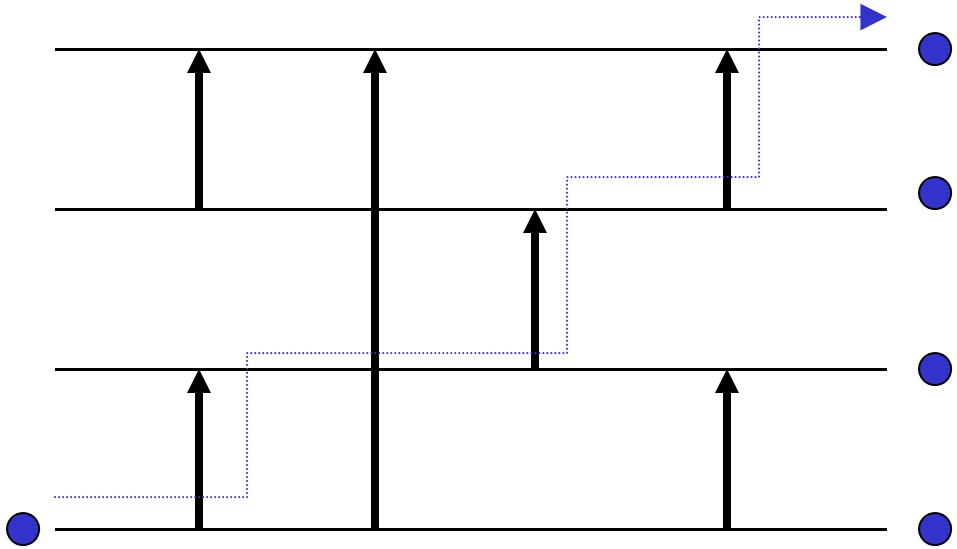

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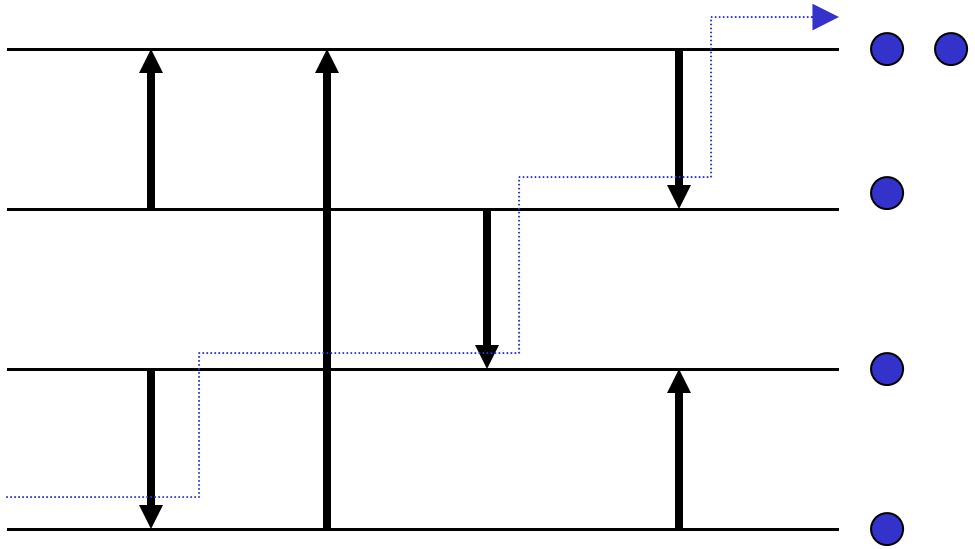

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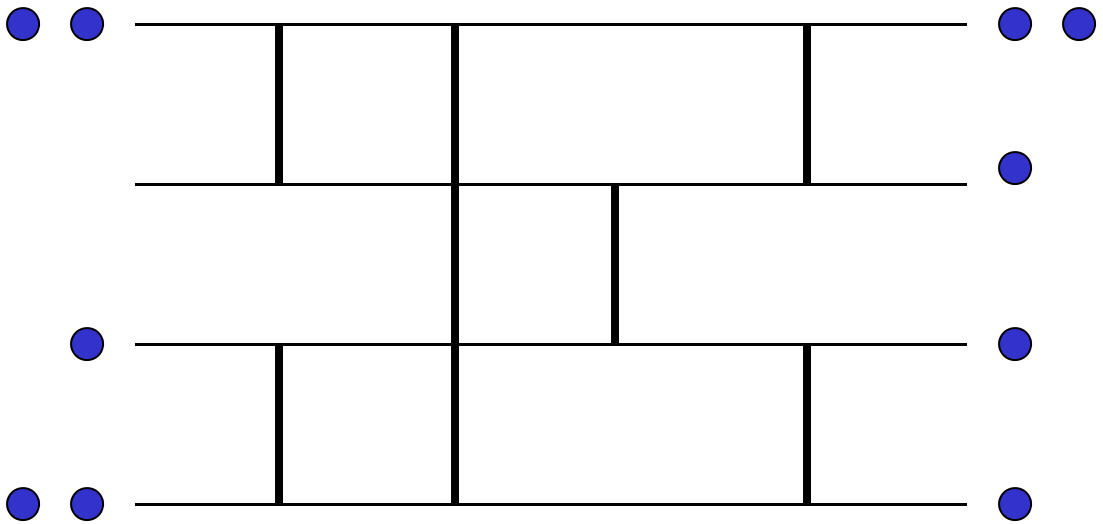
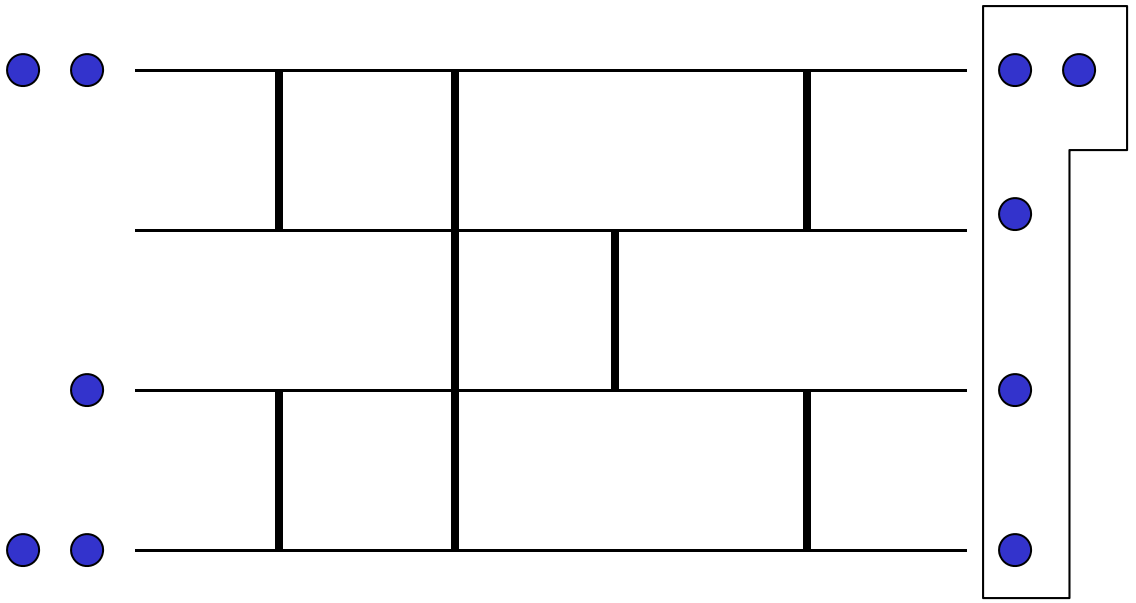


Svi tokeni

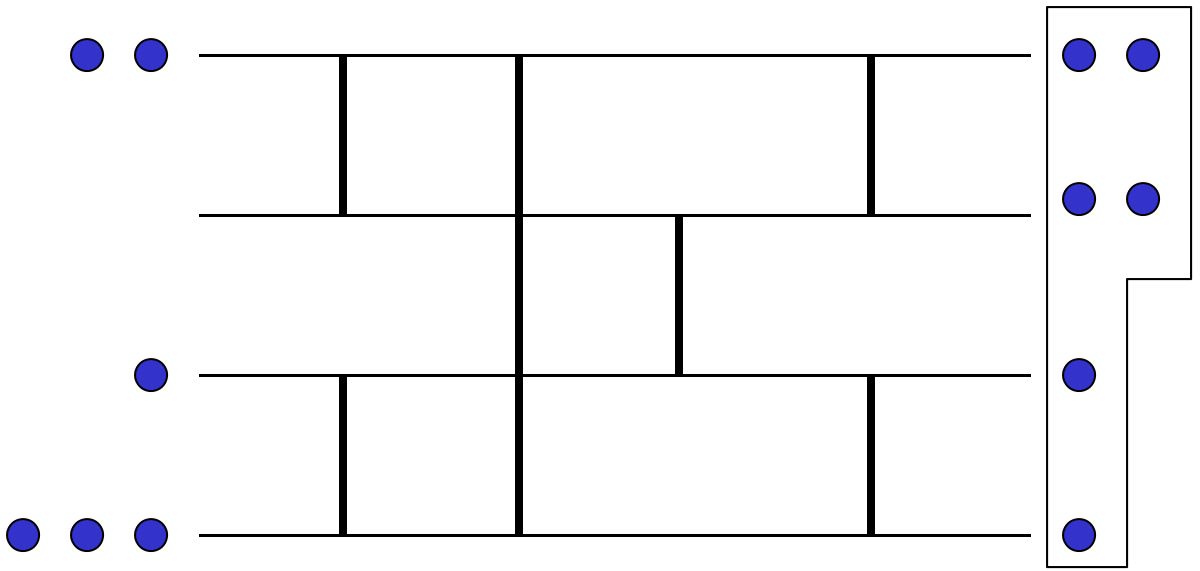
Mreža za brojanje



Svi tokeni

Osobina koraka

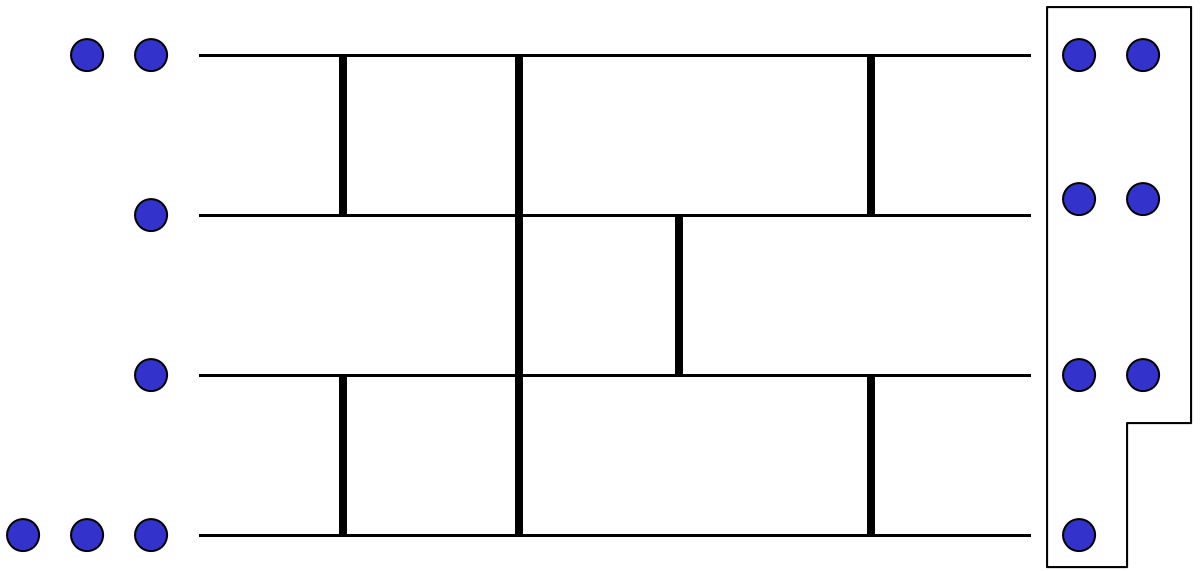
Mreža za brojanje



Sledeći primer

Osobina koraka

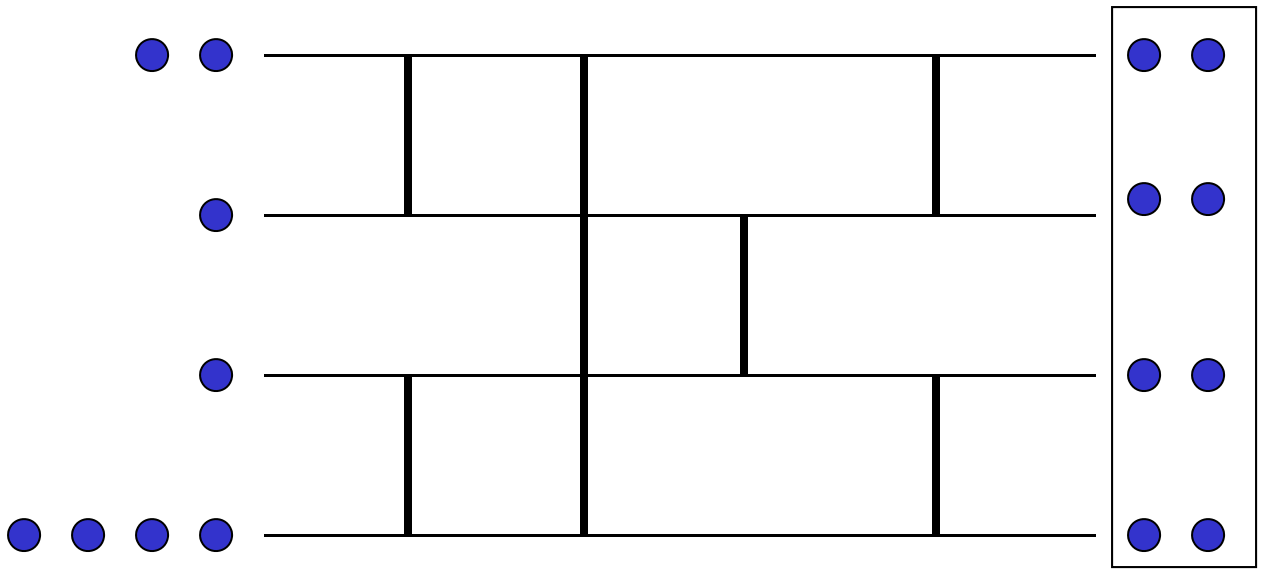
Mreža za brojanje



Sledeći primer

Osobina koraka

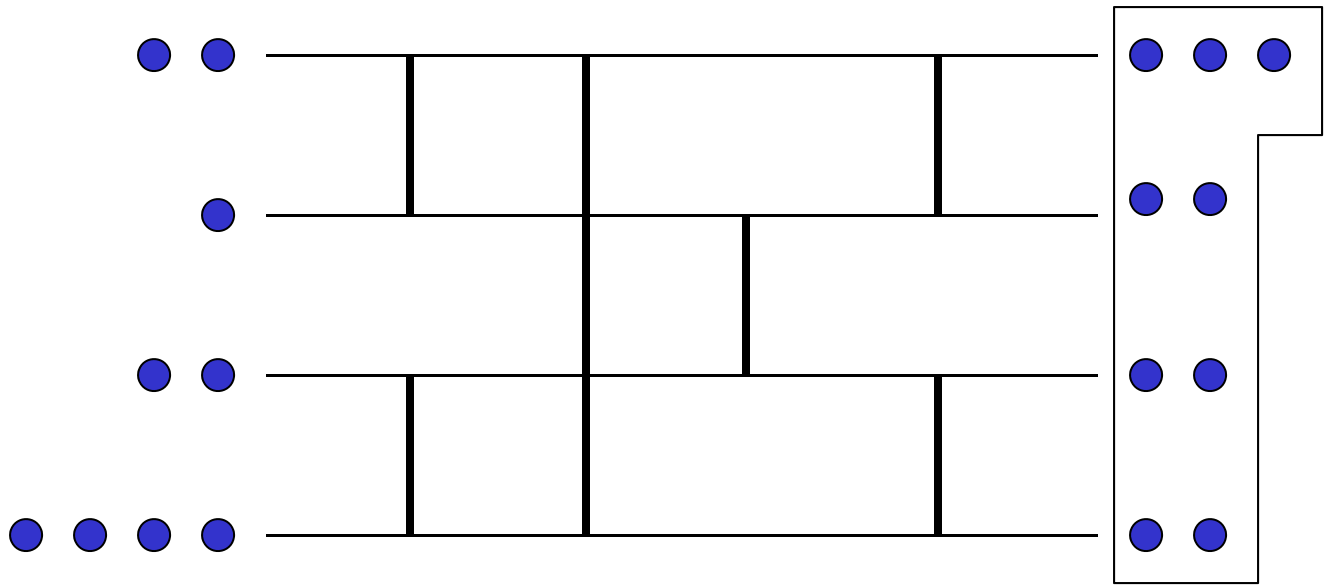
Mreža za brojanje



Sledeći primer

Osobina koraka

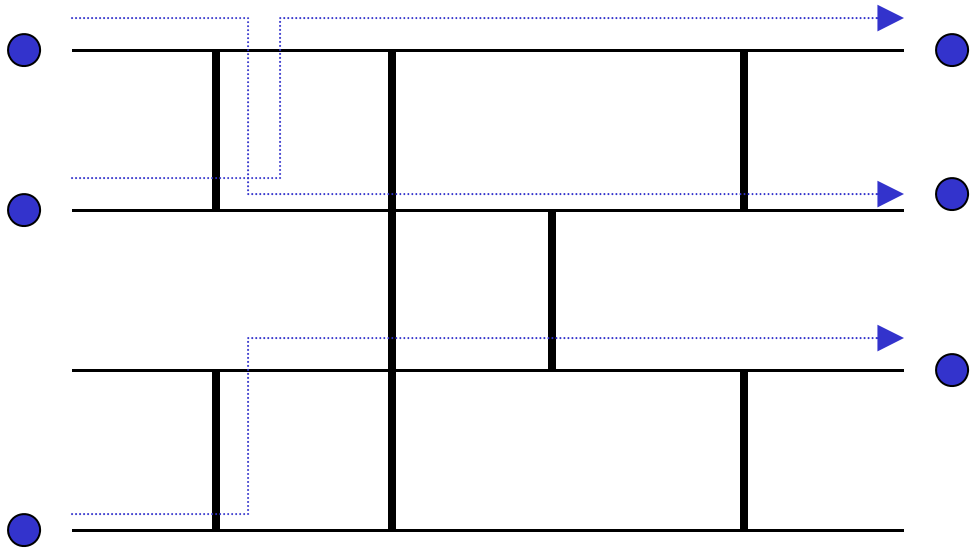
Mreža za brojanje



Osobina koraka

Sledeći primer

Mreža za brojanje



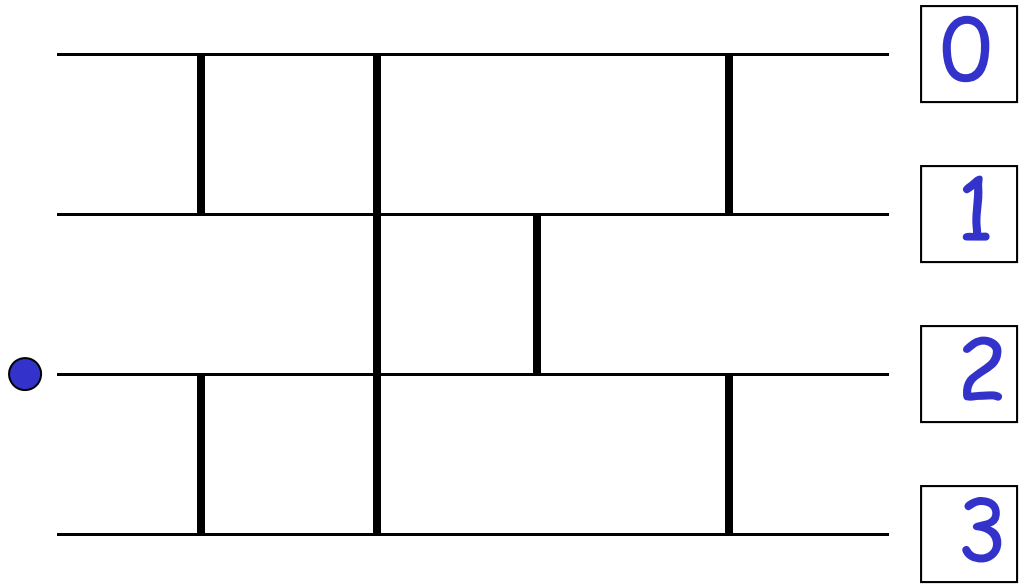
Paralelizam

Mnogi zahtevi za inkrement se obrađuju simultano

Brojanje

Izlaz

Deljene promenljive

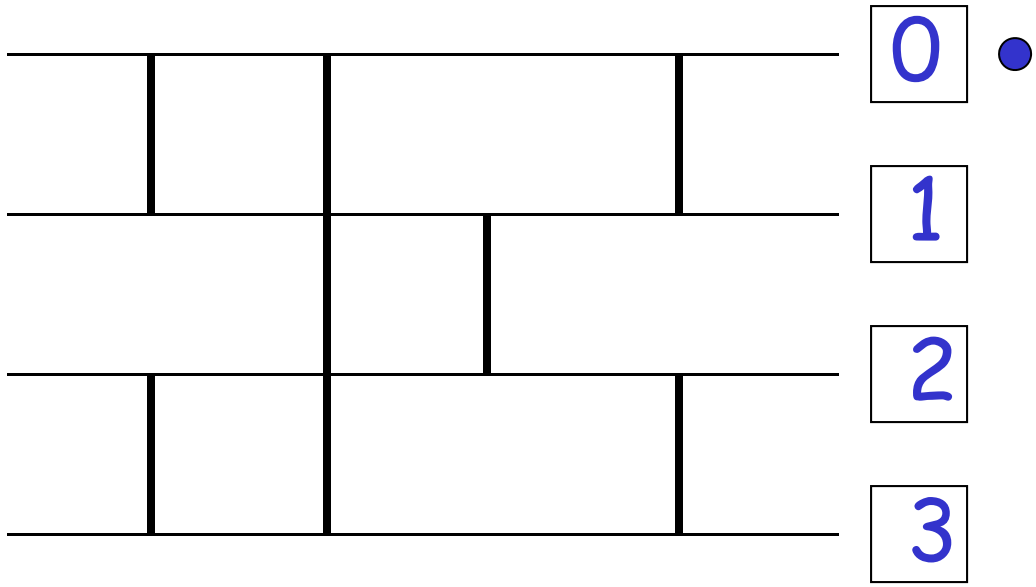


Token = Zahtev za inkrement

Brojanje

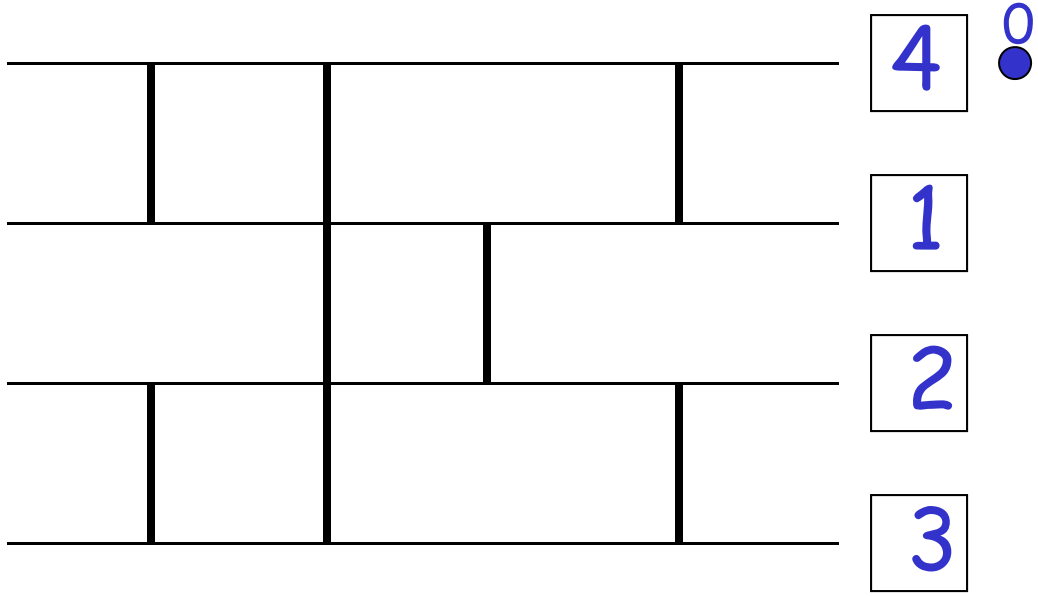
Izlaz

Deljena promenljive



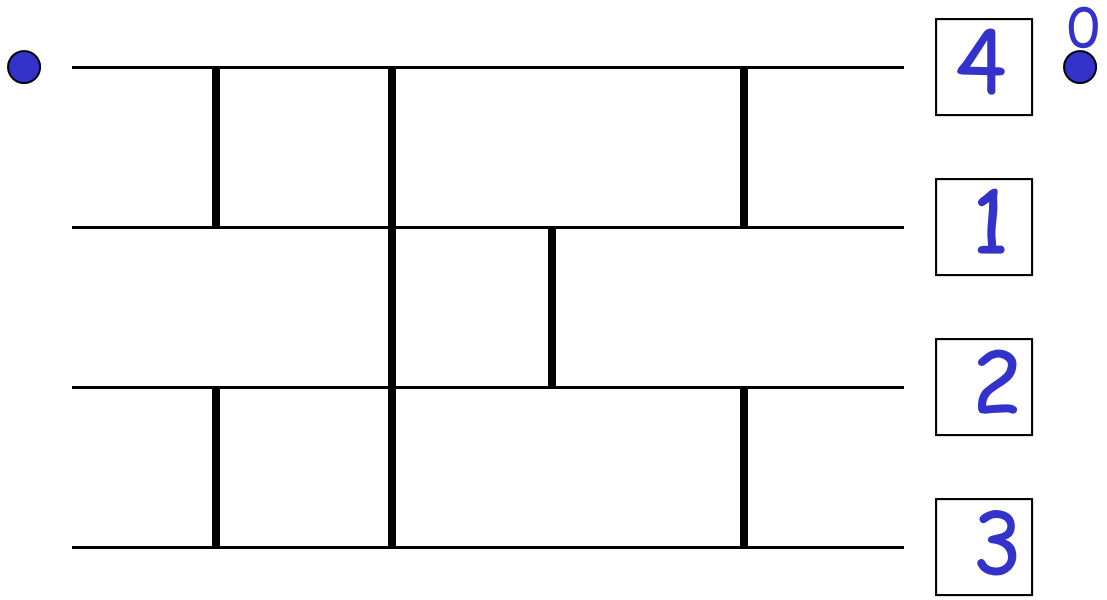
Brojanje

Povratna vrednost



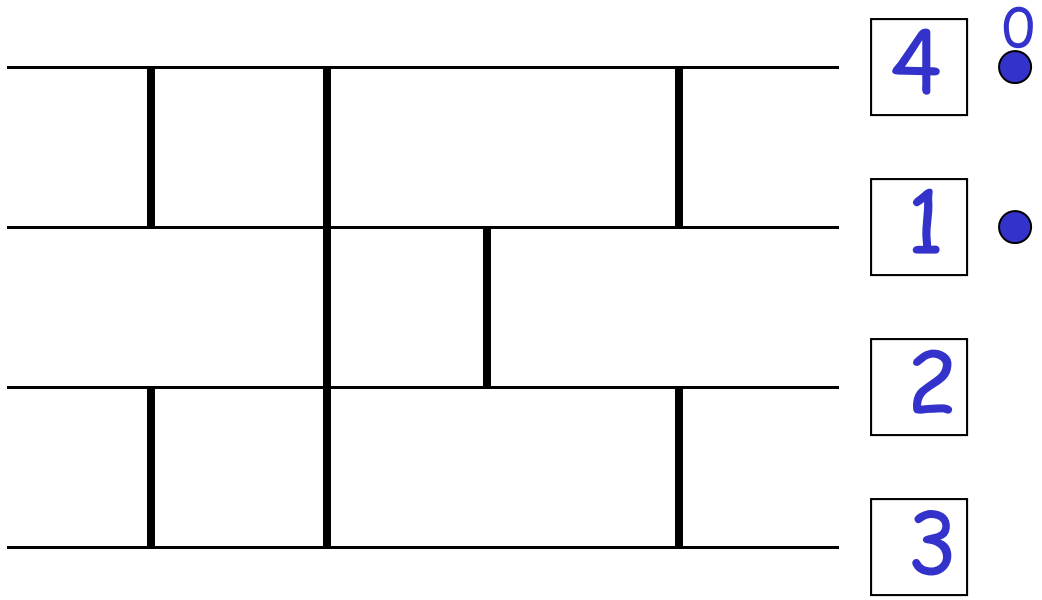
Vrednost deljene promenljive se poveća za 4 (širina izlaza)

Brojanje



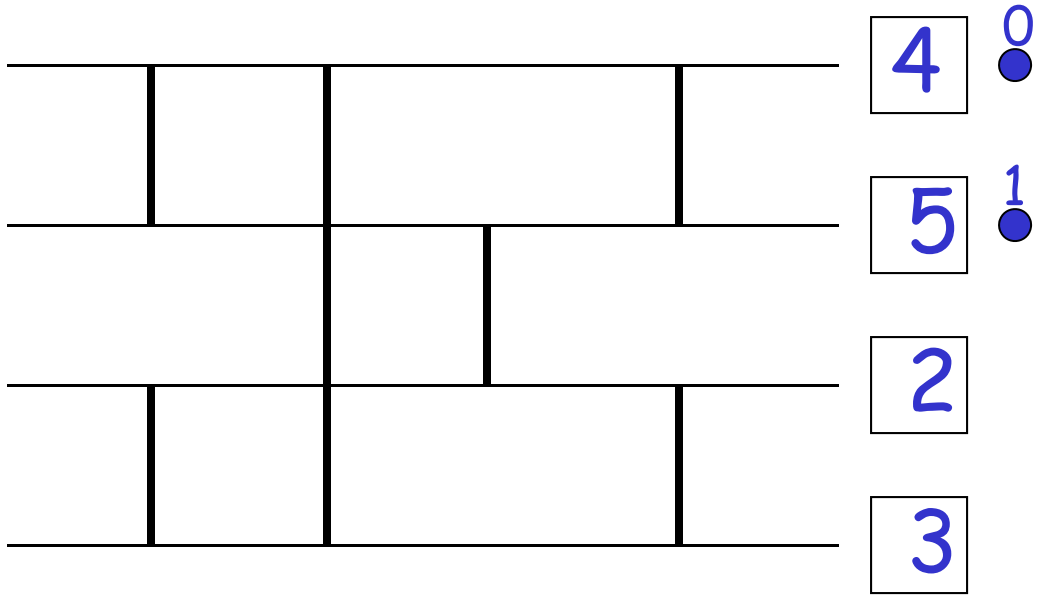
Zahtev za inkrement

Brojanje

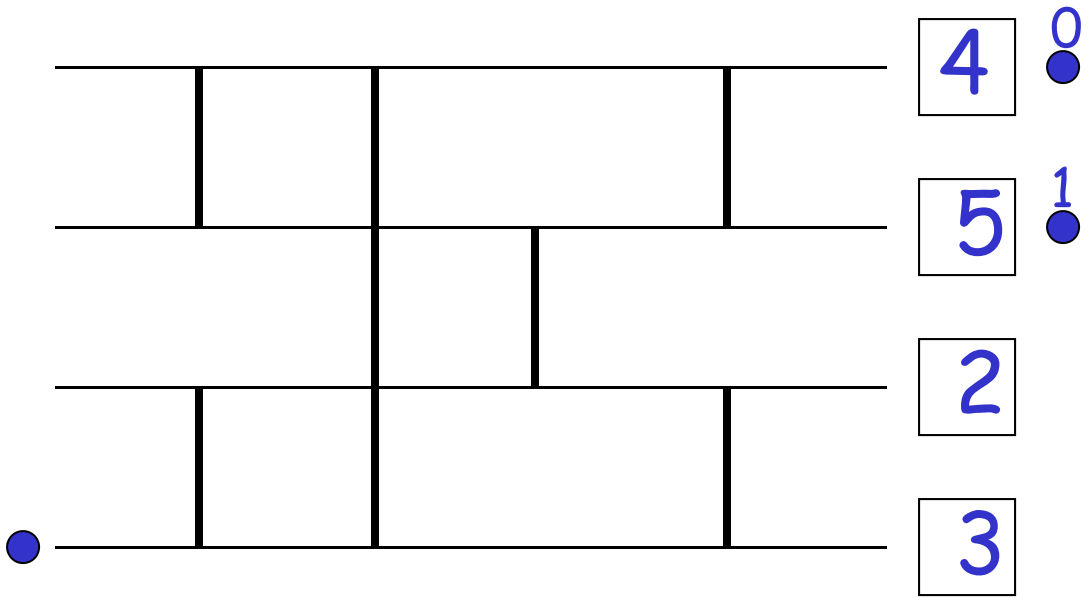


Brojanje

Povratna vrednost



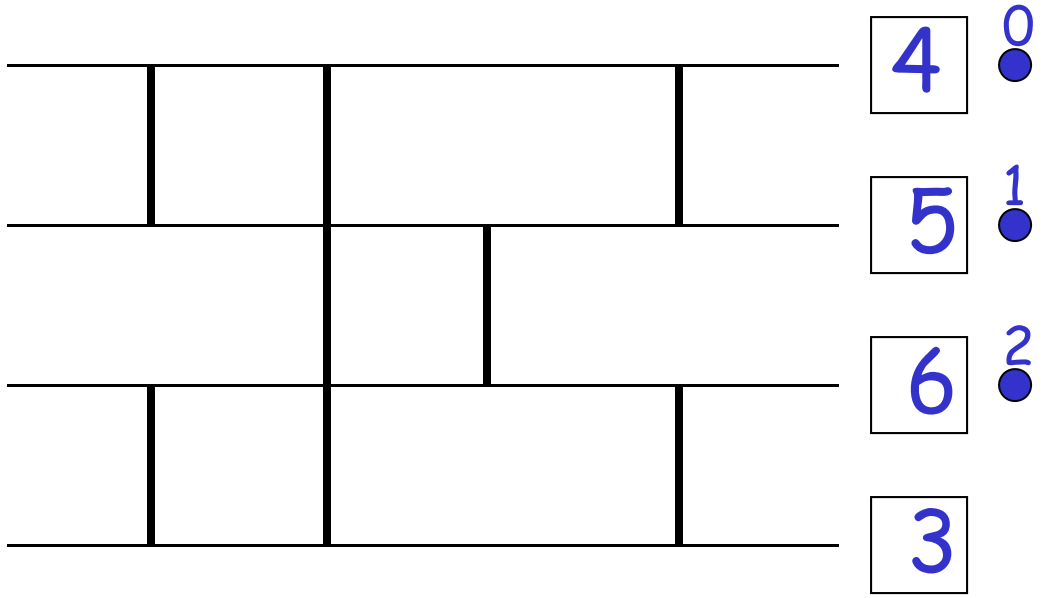
Brojanje



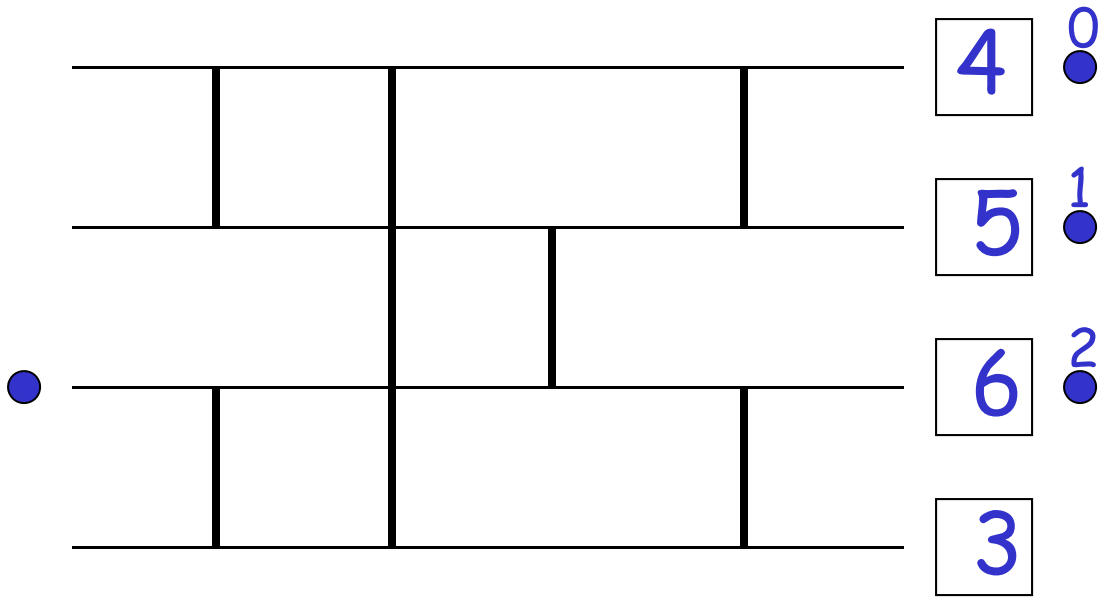
Zahtev za inkrement

Brojanje

Povratna vrednost

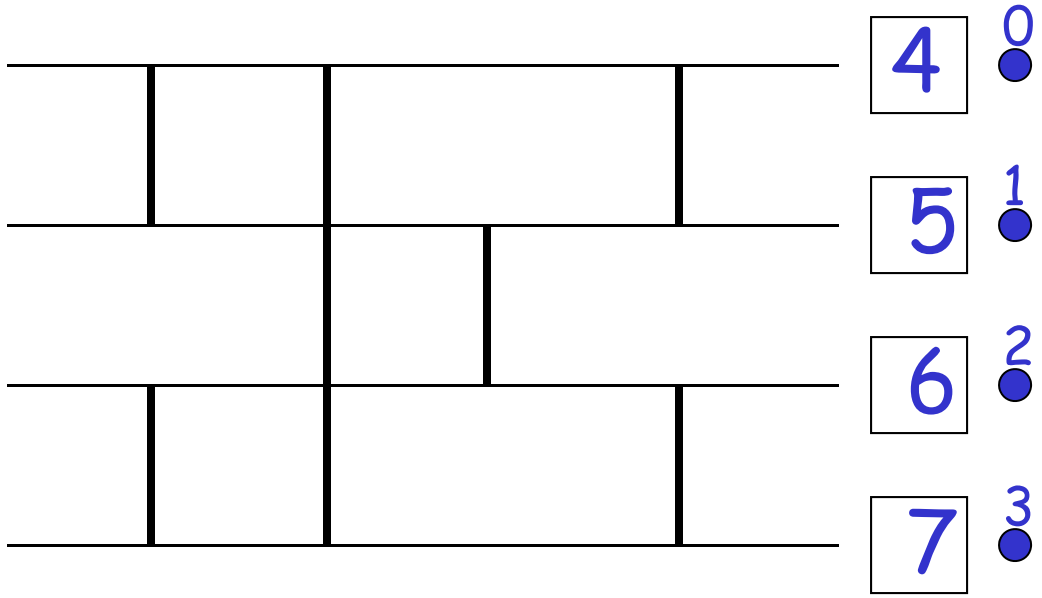


Brojanje

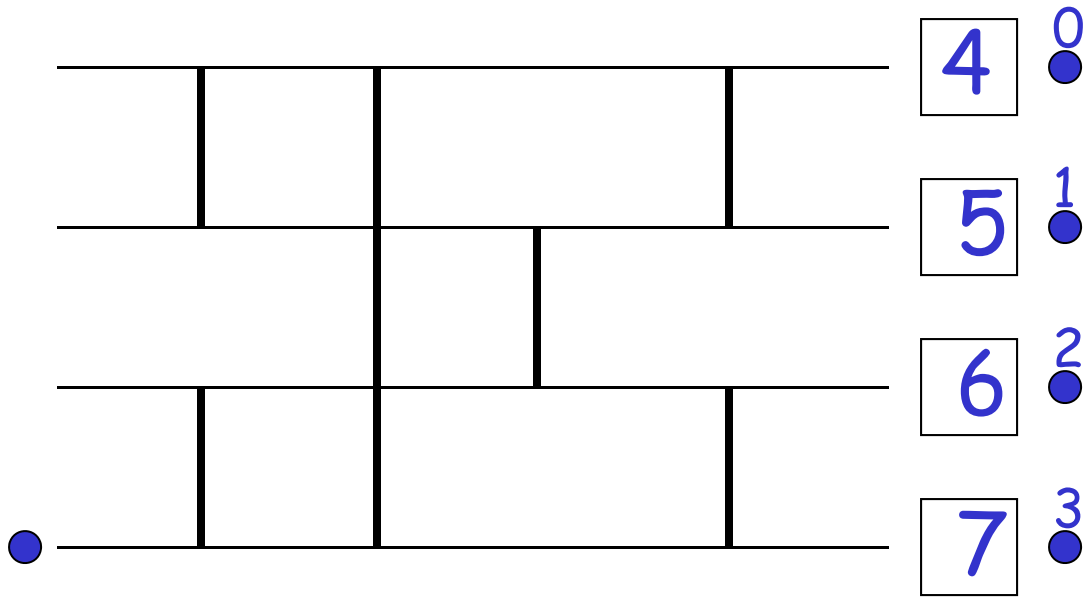


Brojanje

Povratna vrednost

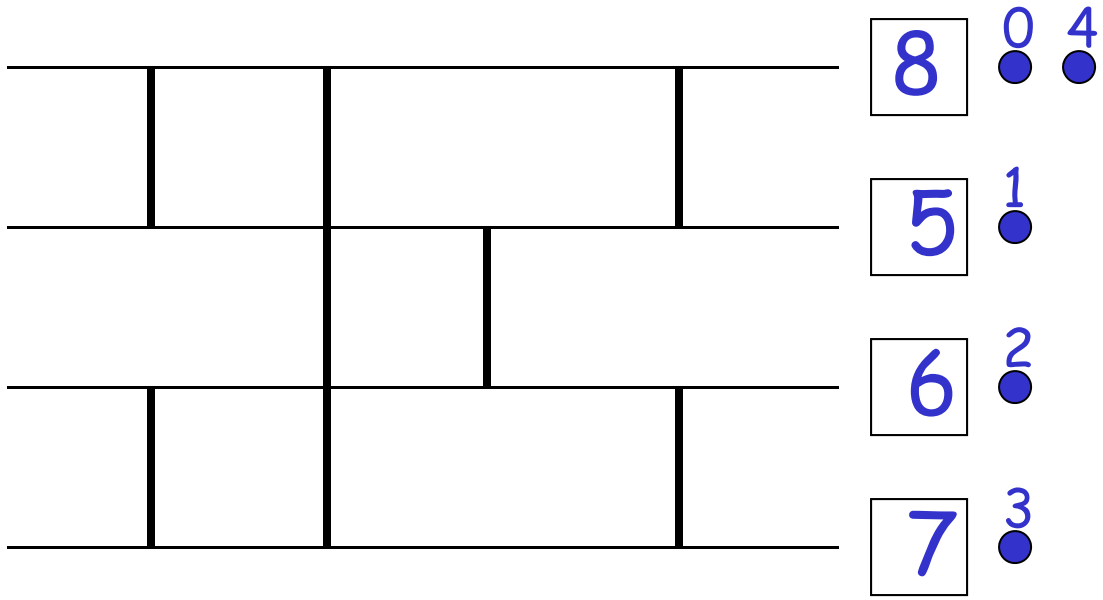


Brojanje

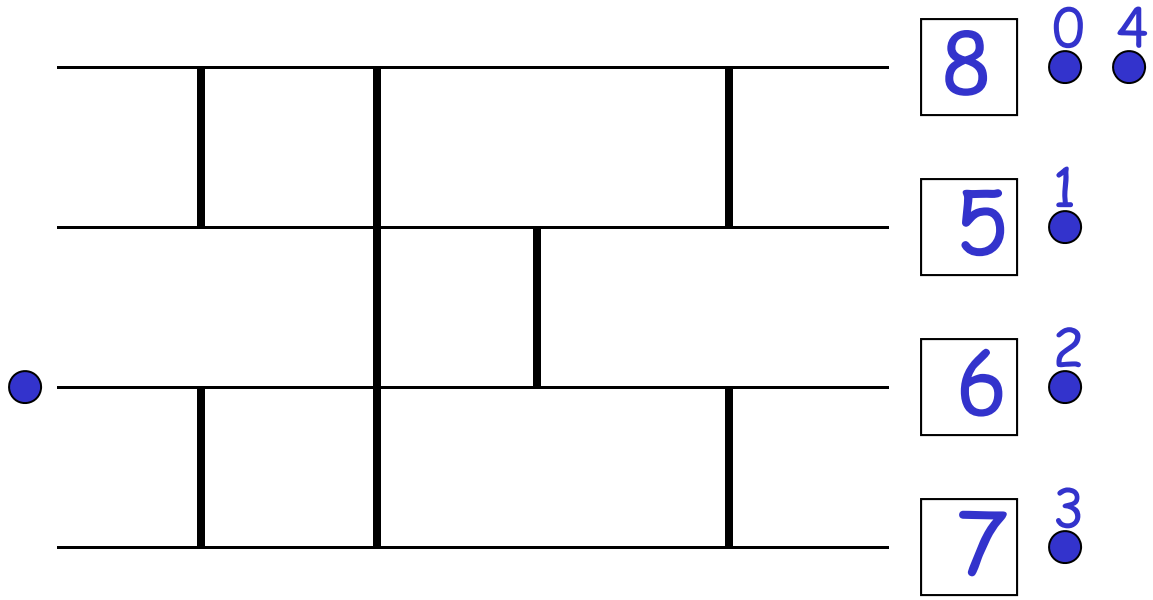


Brojanje

Povratna vrednost

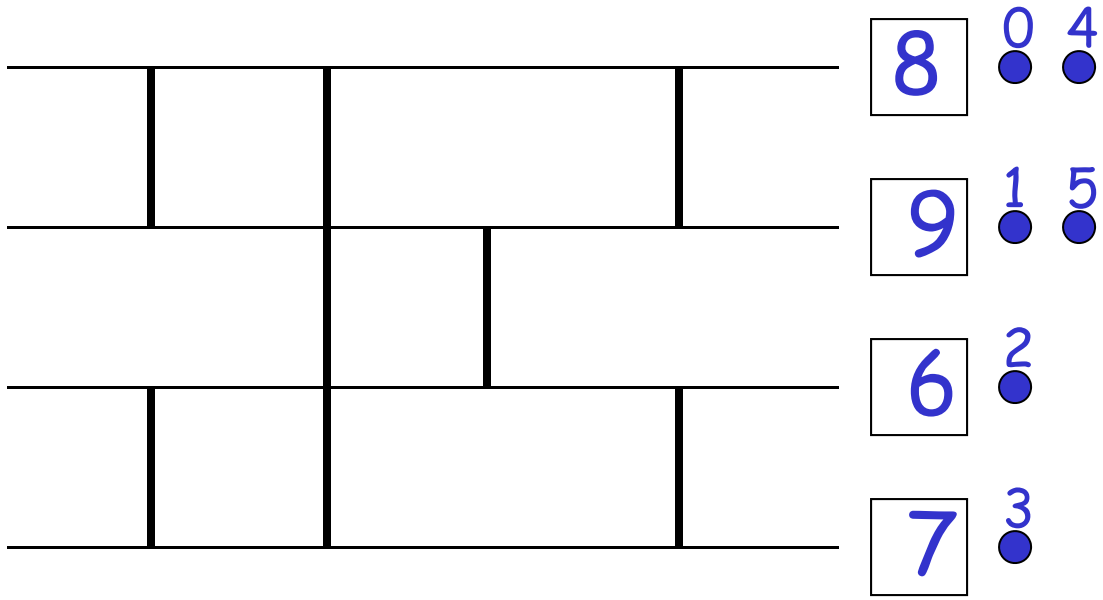


Brojanje

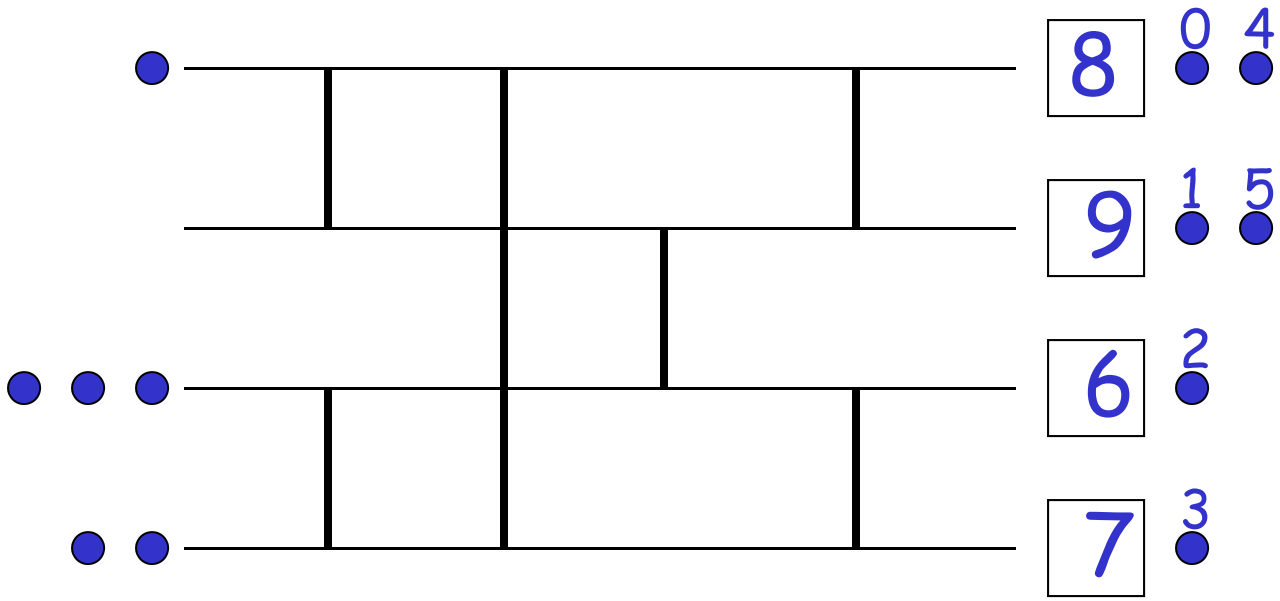


Brojanje

Povratna vrednost



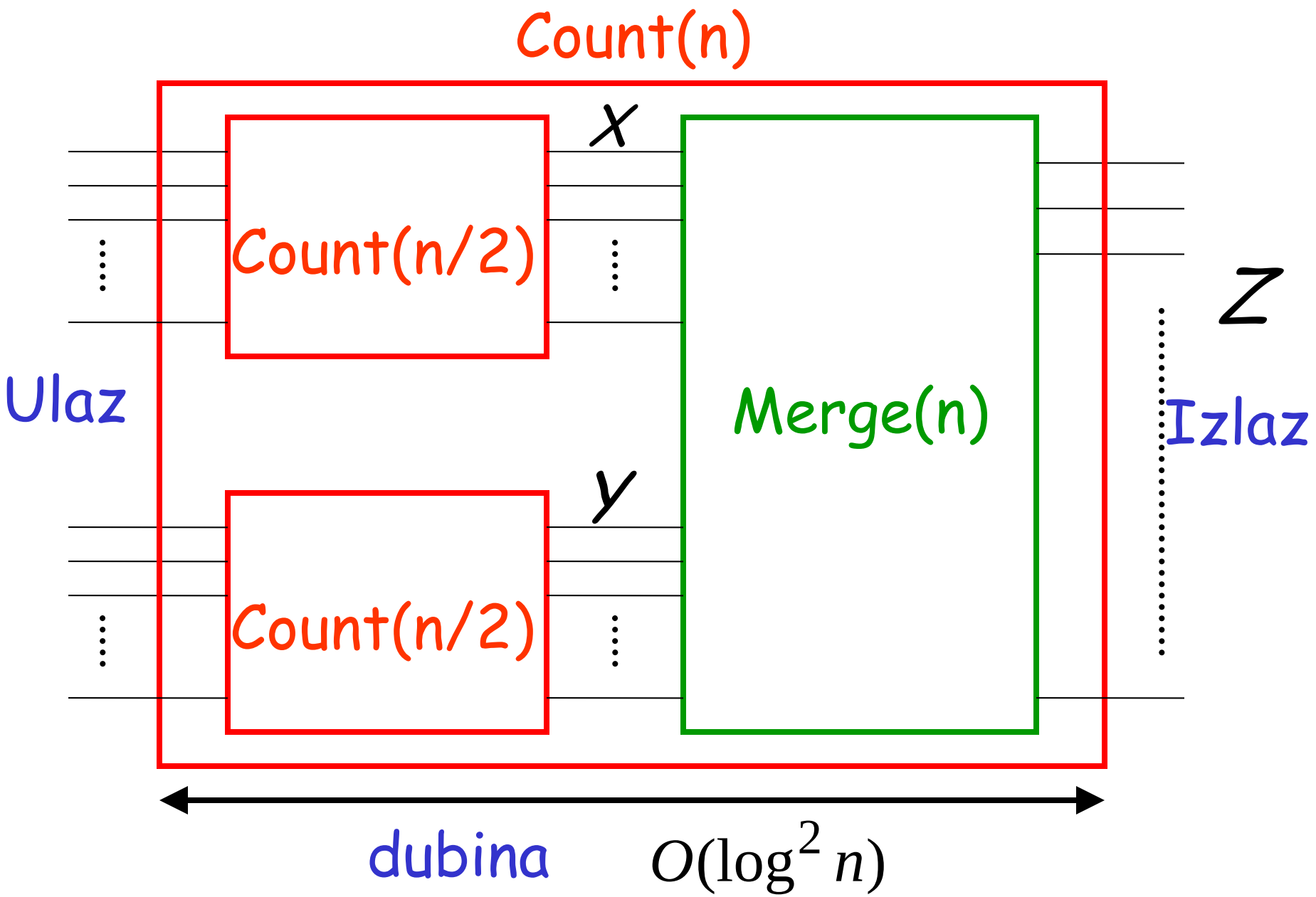
Brojanje

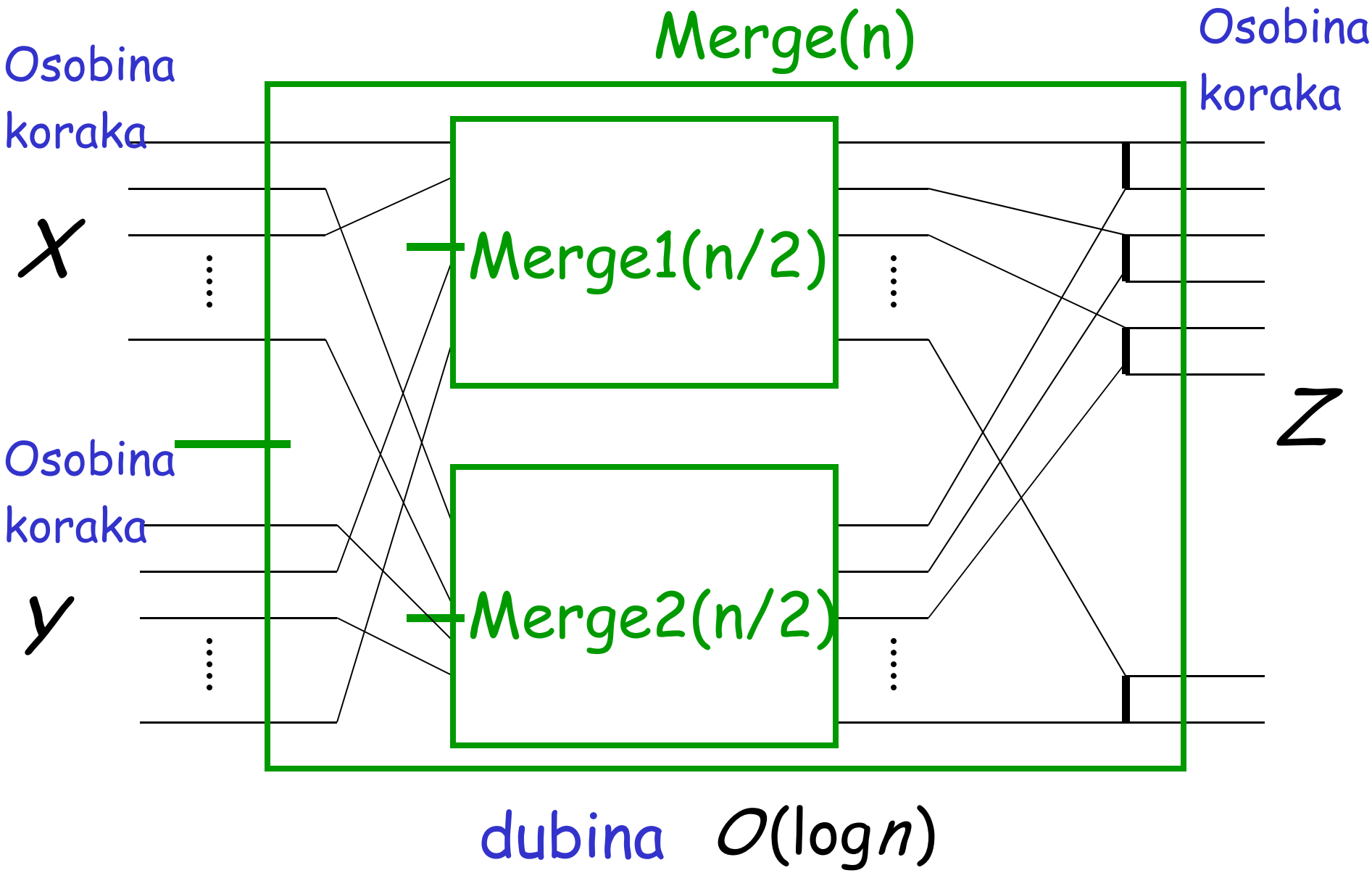


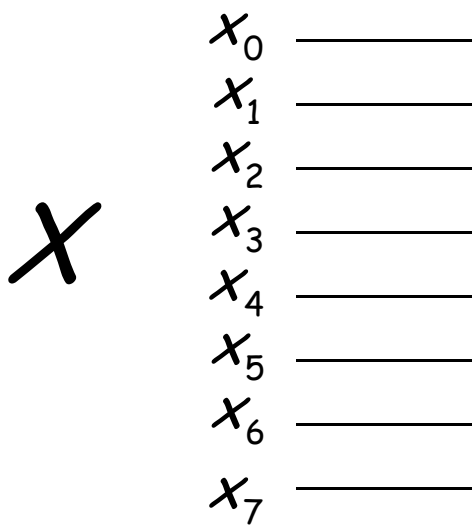
Svi tokeni

Mreža za bitonik brojanje

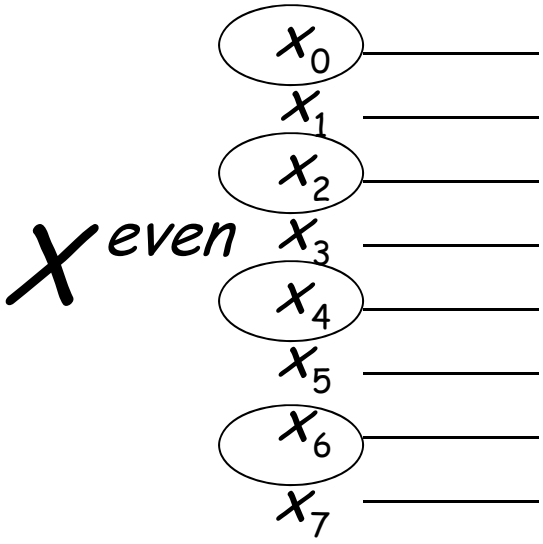
Izomorfna sa Mrežom za bitonik sortiranje


Count(n)
Count(n/2)
Count(n/2)
Merge(n)
x
y
z
Ulaz
Izlaz
dubina
$O(\log^2 n)$

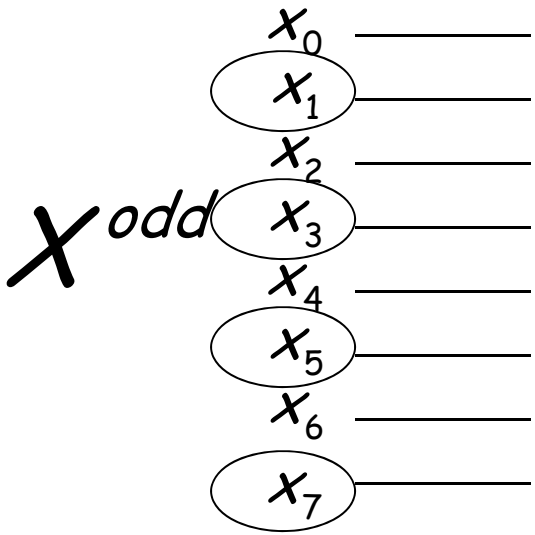

Merge(n)
Osobina koraka
Osobina koraka
x
Merge1(n/2)
Osobina koraka
y
Merge2(n/2)
z
dubina O(log n)

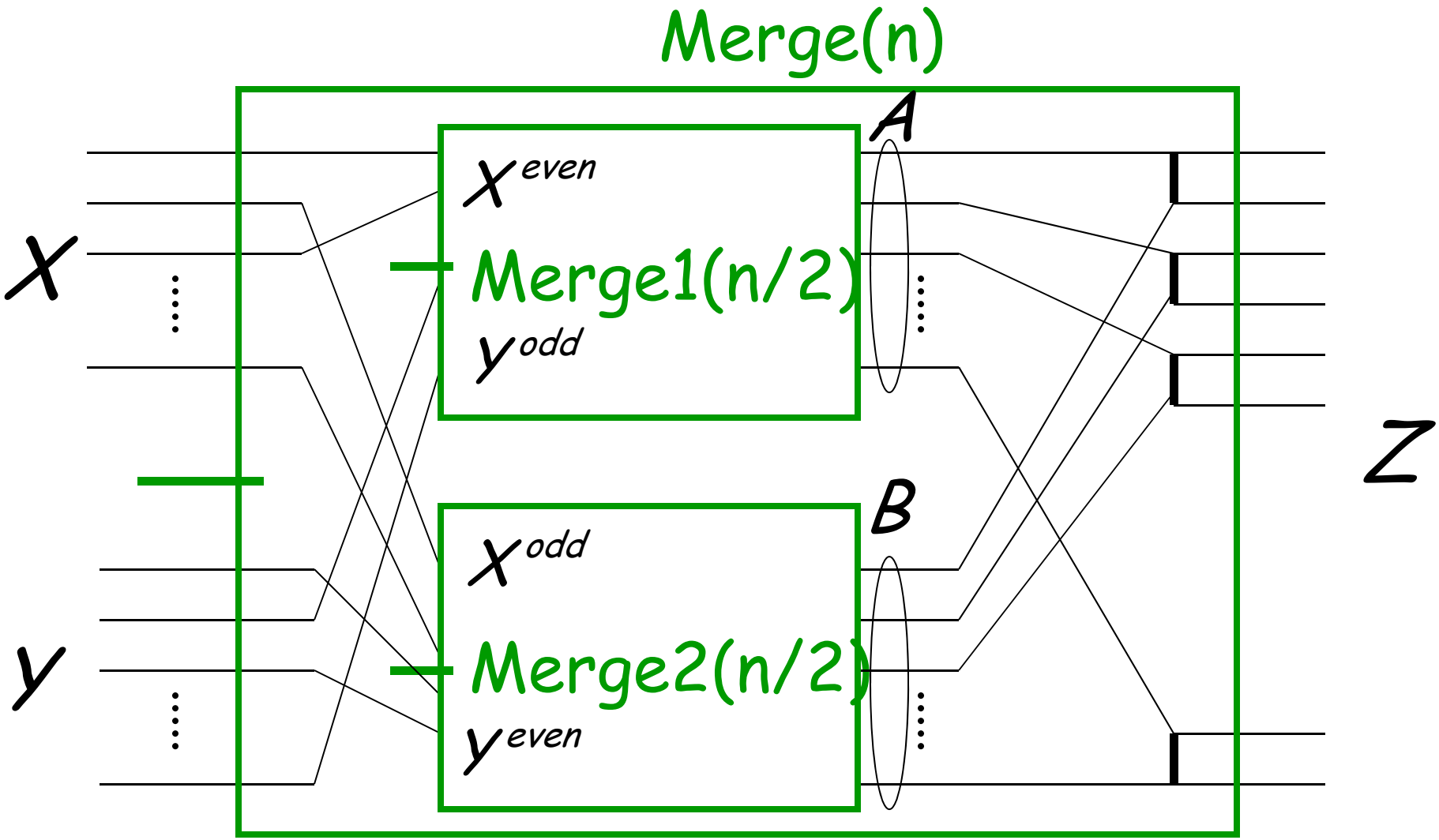


Parne podsekvence



Neparne podsekvence




Merge(n)
X
y
X^{even}
Merge1(n/2)
y^{odd}
X^{odd}
Merge2(n/2)
y^{even}
A
B
Z

Teorema:

merger(n) proizvodi
izlaz Z sa osobinom koraka
ako oba ulaza X i Y imaju osobnu koraka

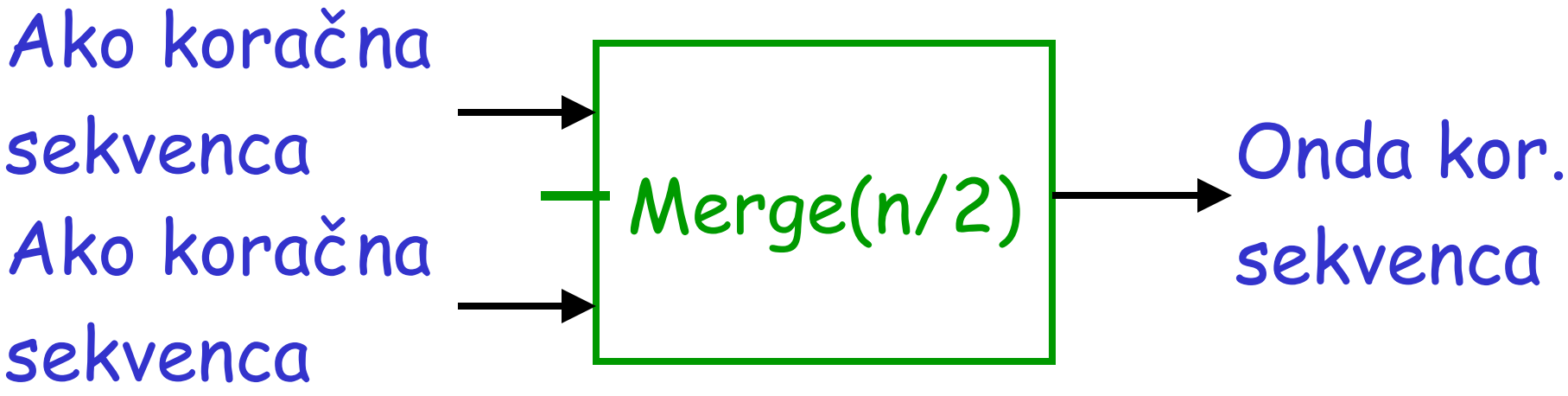
Dokaz:

Dokaz je pomoću indukcije na n

Osnova indukcije:

Za $n = 2$ spajač je balanser i
tvrdnja je trivijalno zadovoljena

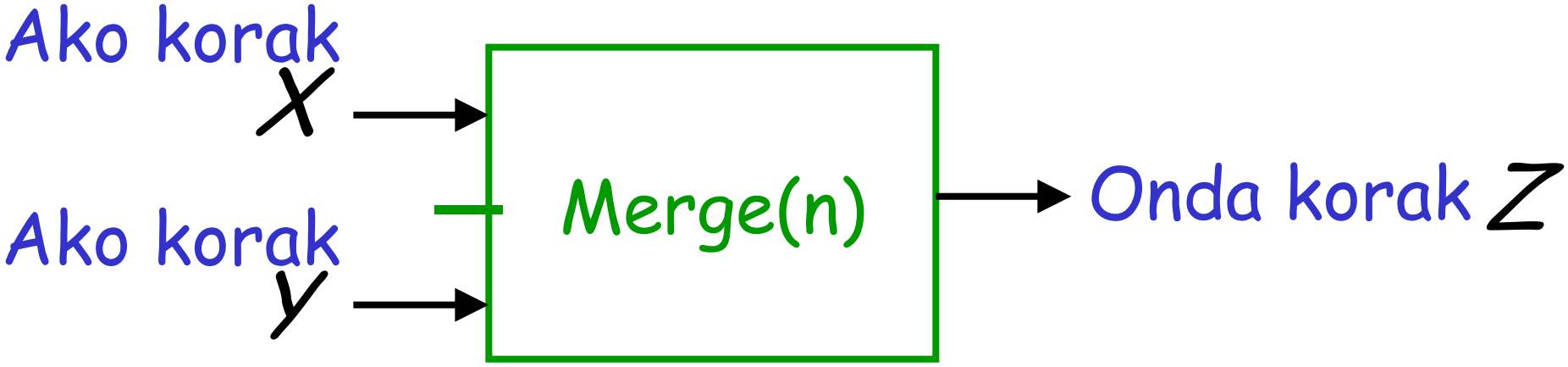
Indukciona hipoteza:



Predpost. da svaki spajač veličine n/2 i manje ispravno obavlja spajanje

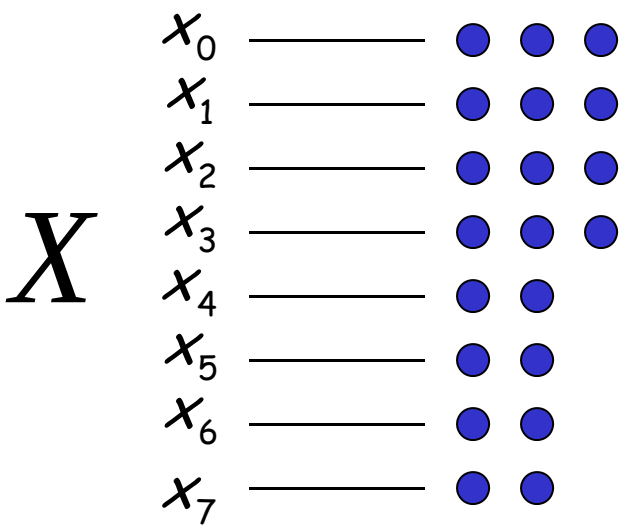
Indukcioni korak:

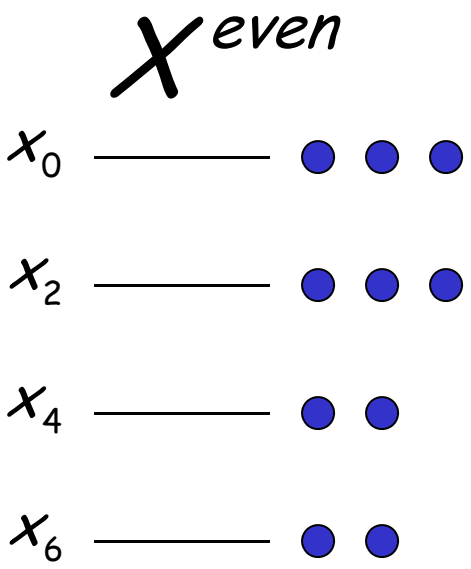
Želimo da pokažemo da je:



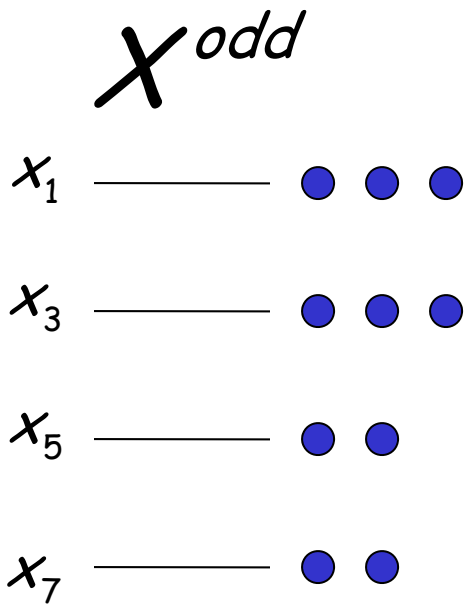
Predpost. da X i Y imaju osobinu koraka.
Onda ćemo pokazati da Z ima osobinu koraka

Tokeni

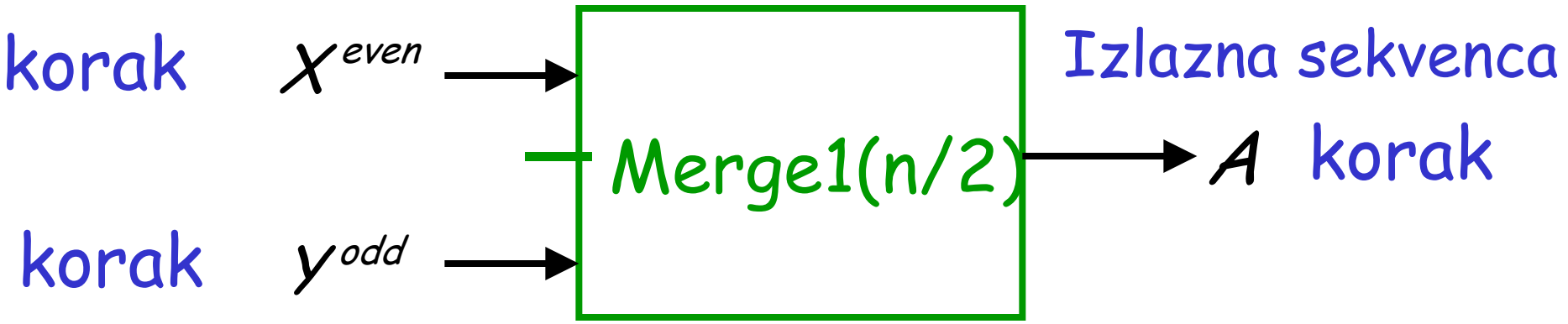


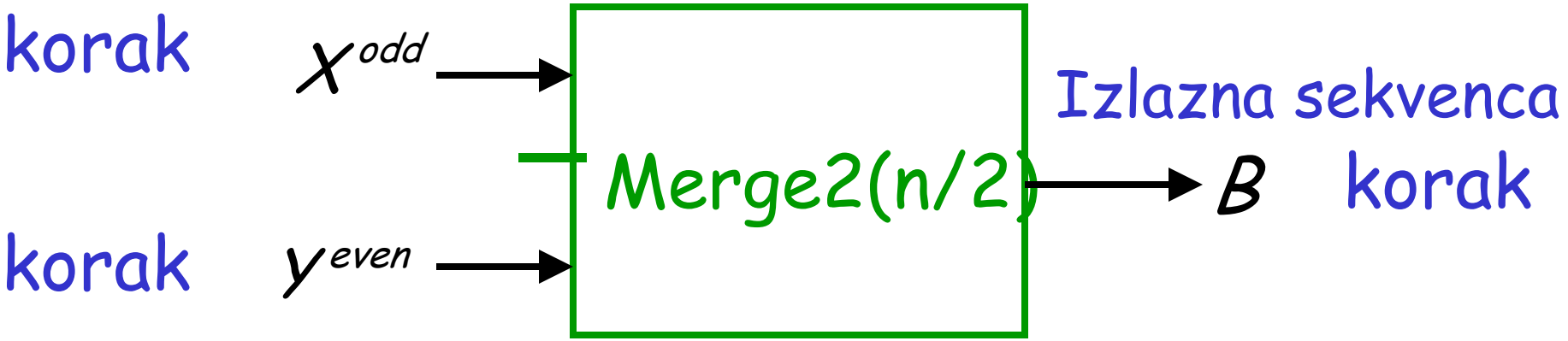


Ako X ima osobinu koraka onda X^{even} i X^{odd} imaju osobinu koraka



Zato, iz indukcione hipoteze:





Prvo, pokažimo: $-1 \leq |A| - |B| \leq 1$

Gde $|A|$ označava ukupan broj tokena u sekevnci A

Spajač 1

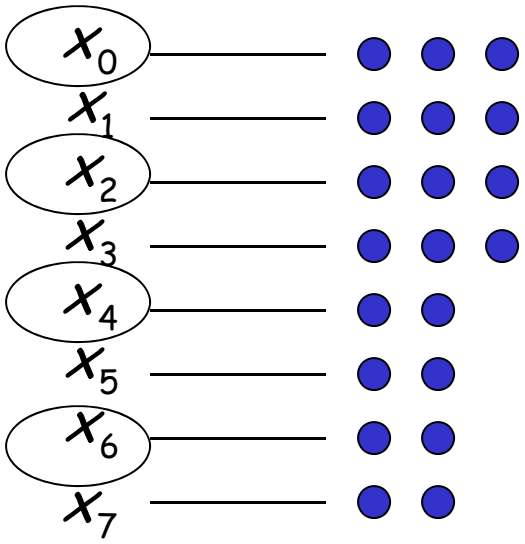
Imamo: $|A| = |X^{even}| + |Y^{odd}|$

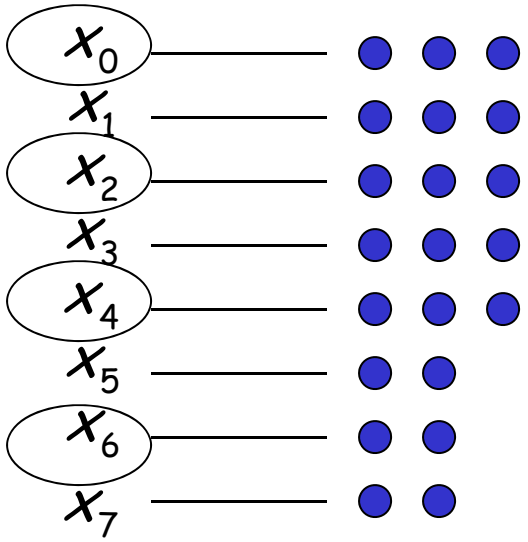
Spajač 2

$|B| = |Y^{even}| + |X^{odd}|$

Pošto X ima osobinu koraka:

$|X^{even}| = |X^{odd}|$ ili $|X^{even}| = |X^{odd}| + 1$





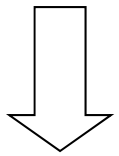
Zato je: $0 \le |X^{even}| - |X^{odd}| \le 1$

Slično: $0 \le |Y^{even}| - |Y^{odd}| \le 1$

$$0 \le |X^{even}| - |X^{odd}| \le 1$$

$$0 \le |Y^{even}| - |Y^{odd}| \le 1$$

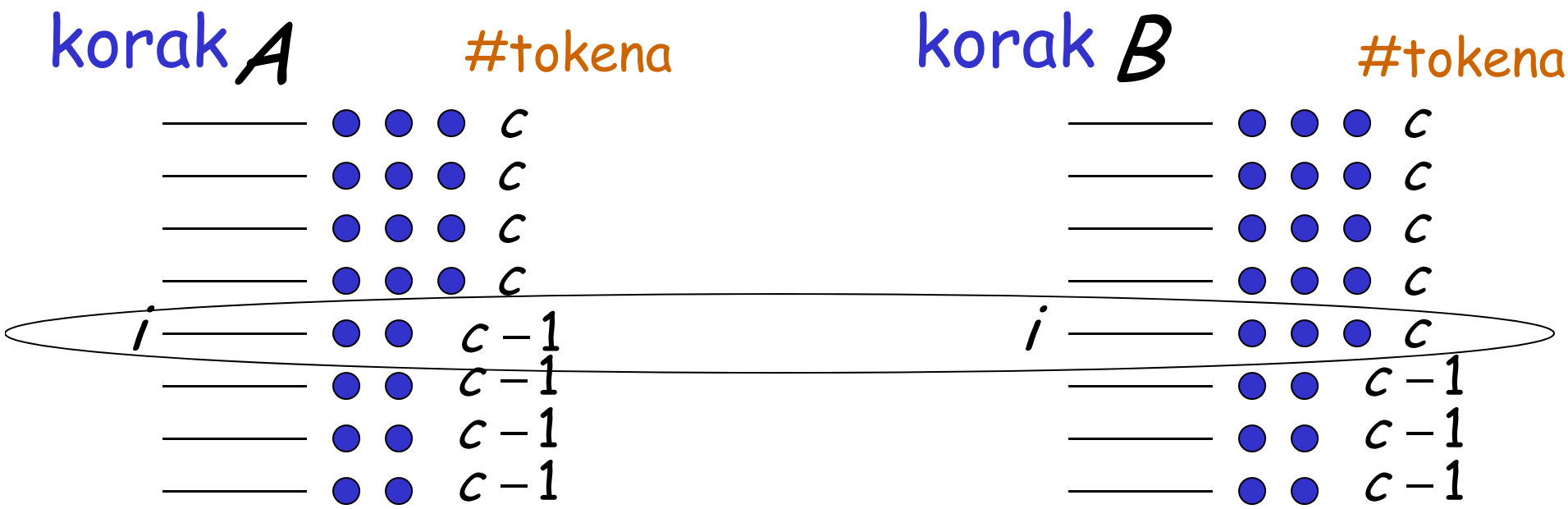
$$|A| - |B| = (|X^{even}| - |X^{odd}|) + (|Y^{odd}| - |Y^{even}|)$$



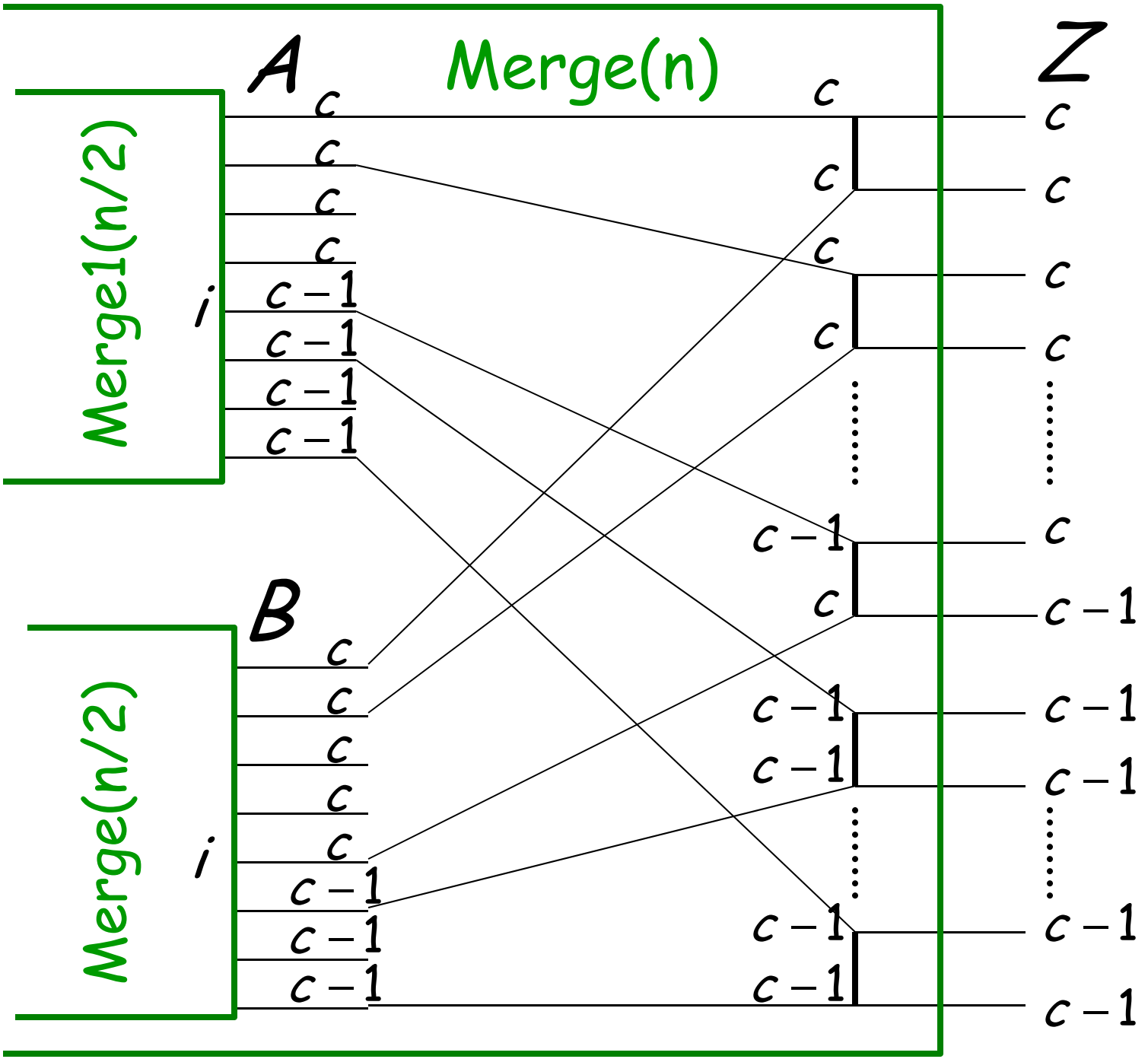
$$-1 \le |A| - |B| \le 1$$

Sada, pokažimo da Z ima osobinu koraka

$$-1 \leq |A| - |B| \leq 1$$



Postoji najviše jedna žica i gde se dve sekvence razlikuju



Osobina koraka